

Disambiguation and Layout-Sensitive Syntax

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Disambiguation and Layout-Sensitive Syntax

Syntax Definition Summary

Derivations

- Generating sentences and trees from context-free grammars

Ambiguity

Declarative Disambiguation Rules

- Associativity and priority

Grammar Transformations

- Eliminating ambiguity by transformation

Layout-Sensitive Syntax

- Disambiguation using layout constraints

Structure

Syntax = Structure

```
module structure
```

```
imports Common
```

```
context-free start-symbols Exp
```

```
context-free syntax
```

```
Exp.Var = ID
```

```
Exp.Int = INT
```

```
Exp.Add = Exp "+" Exp
```

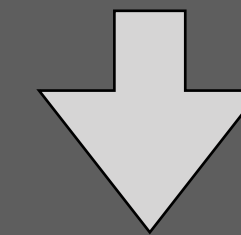
```
Exp.Fun = "function" "(" {ID ","}* ")" "{" Exp "}"
```

```
Exp.App = Exp "(" {Exp ","}* ")"
```

```
Exp.Let = "let" Bnd* "in" Exp "end"
```

```
Bnd.Bnd = ID "=" Exp
```

```
let  
  inc = function(x) { x + 1 }  
in  
  inc(3)  
end
```



```
Let(  
  [ Bnd(  
    "inc"  
    , Fun(["x"], Add(Var("x"), Int("1")))  
  )  
  ]  
  , App(Var("inc"), [Int("3")])  
)
```

Token = Character

```
module structure
```

```
imports Common
```

```
context-free start-symbols Exp
```

```
context-free syntax
```

```
Exp.Var = ID
```

```
Exp.Int = INT
```

```
Exp.Add = Exp "+" Exp
```

```
Exp.Fun = "function" "(" {ID ","}* ")" "{" Exp "}"
```

```
Exp.App = Exp "(" {Exp ","}* ")"
```

```
Exp.Let = "let" Bnd* "in" Exp "end"
```

```
Bnd.Bnd = ID "=" Exp
```

```
let  
  inc = function(x) { x + 1 }  
in  
  inc(3)  
end
```

```
module Common
```

```
lexical syntax
```

```
ID = [a-zA-Z] [a-zA-Z0-9]*
```

```
INT = [\-]? [0-9]+
```

Lexical Syntax = Context-Free Syntax
(But we don't care about structure of lexical syntax)

Literal = Non-Terminal

```
module structure
```

```
imports Common
```

```
context-free start-symbols Exp
```

```
context-free syntax
```

```
Exp.Var = ID
```

```
Exp.Int = INT
```

```
Exp.Add = Exp "+" Exp
```

```
Exp.Fun = "function" "(" {ID ","}* ")" "{" Exp "}"
```

```
Exp.App = Exp "(" {Exp ","}* ")"
```

```
Exp.Let = "let" Bnd* "in" Exp "end"
```

```
Bnd.Bnd = ID "=" Exp
```

```
let
```

```
  inc = function(x) { x + 1 }
```

```
in
```

```
  inc(3)
```

```
end
```

```
syntax
```

```
"+" = [\43]
```

```
"function" = [\102] [\117] [\110] [\99]  
            [\116] [\105] [\111] [\110]
```

```
"{" = [\123]
```

```
"}" = [\125]
```

```
"(" = [\40]
```

```
"," = [\44]
```

```
")" = [\41]
```

```
"let" = [\108] [\101] [\116]
```

```
"in" = [\105] [\110]
```

```
"end" = [\101] [\110] [\100]
```

```
"=" = [\61]
```

Layout = Whitespace & Comments

```
module Common
```

```
lexical syntax
```

```
LAYOUT = [\ \t\n\r]
```

```
LAYOUT = "/*" InsideComment* "*/"
```

```
InsideComment = ~[\*]
```

```
InsideComment = CommentChar
```

```
CommentChar = [\*]
```

```
LAYOUT = "//" ~[\n\r]* NewLineEOF
```

```
NewLineEOF = [\n\r]
```

```
NewLineEOF = EOF
```

```
let  
  inc = function(x) { x + 1 }  
in  
  // function application  
  inc /* function position */ (  
    3 // argument list  
  )  
end
```

Layout = (Almost) Everywhere

```
module Common
```

```
lexical syntax
```

```
LAYOUT = [\ \t\n\r]
```

```
LAYOUT = "/*" InsideComment* "*/"
```

```
InsideComment = ~[\*]
```

```
InsideComment = CommentChar
```

```
CommentChar = [\*]
```

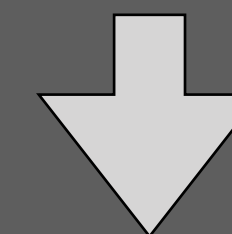
```
LAYOUT = "//" ~[\n\r]* NewLineEOF
```

```
NewLineEOF = [\n\r]
```

```
NewLineEOF = EOF
```

```
let  
  inc = function(x) { x + 1 }  
in  
  // function application  
  inc /* function position */ (  
    3 // argument list  
  )  
end
```

```
Exp.App = Exp "(" {Exp ","}* ")"
```



```
Exp-CF.App = Exp-CF LAYOUT?-CF "(" LAYOUT?-CF {Exp-CF ","}* LAYOUT?-CF ")"
```


Extension

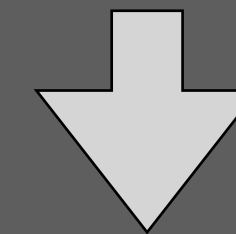
Language Composition \Rightarrow Grammar Composition

```
module extension
imports functional query
context-free start-symbols Exp
context-free syntax
  Exp = Query
  Cond = Exp
```

```
module functional
imports Common
context-free syntax
  Exp = <(<Exp>)> {bracket}
  ...
```

```
module query
imports Common
context-free syntax
  Query.Query = <
    select <QID*> from <QID*> where <Cond>
  >
  Cond.And = <<Cond> and <Cond>> {left}
  Cond.Eq = <<Cond> == <Cond>> {non-assoc}
```

```
let
  select = 1
  fs = select f from A where test f = select
in
  print fs
```



```
Let(
  [ Bnd("select", Int("1"))
  , Bnd(
    "fs"
    , Query(
      ["f"]
      , ["A"]
      , Eq(App(Var("test"), Var("f")), Var("select"))
    )
  )
]
, App(Var("print"), Var("fs"))
)
```

Parsing = Formatting⁻¹

Parsing = Formatting⁻¹

context-free syntax

Exp.Var = <<ID>>

Exp.Int = <<INT>>

Exp.Add = <<Exp> + <Exp>>

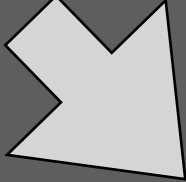
Exp.Fun = <
function(<{ID " , "}*>){
 <Exp>
}
>

Exp.App = <<Exp>(<{Exp " , "}*>)>

Exp.Let = <
let
 <{Bnd "\n"}*>
in
 <Exp>
end
>

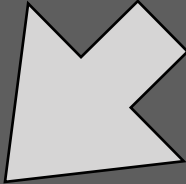
Bnd.Bnd = <<ID> = <Exp>>

```
let
  inc = function(x) { x + 1 }
in
  inc(3)
end
```



```
Let(
  [ Bnd(
    "inc"
    , Fun(["x"], Add(Var("x"), Int("1")))
  )
  , App(Var("inc"), [Int("3")])
)
```

```
let
  inc = function(x){
    x + 1
  }
in
  inc(3)
end
```



Completion = Rewrite(Incomplete Structure)

```
class A {  
    public int m() {  
        int x;  
        x = $Exp;  
        return $Exp;  
    }  
}
```

+Add \$Exp + \$Exp
+Sub
+Mul
+Lt
+VarRef

```
class A {  
    public int m() {  
        int x;  
        x = $Exp + $Exp;  
        return $Exp + $Exp;  
    }  
}
```

+Add \$Exp + \$Exp
+Sub
+Mul
+Lt
+VarRef

```
class A {  
    public int m() {  
        int x;  
        x = 21 + $Exp;  
        return x;  
    }  
}
```

+Add (\$Exp + \$Exp)
+Sub
+Mul
+Lt
+VarRef

```
class A {  
    public int m() {  
        int x;  
        x = 21 + 21;  
        return x;  
    }  
}
```

Context-Free Grammars

Context-Free Grammars

Terminals

- Basic symbols from which strings are formed

Nonterminals

- Syntactic variables that denote sets of strings

Start Symbol

- Denotes the nonterminal that generates strings of the languages

Productions

- $A = X \dots X$
- Head/left side (A) is a nonterminal
- Body/right side ($X \dots X$) zero or more terminals and nonterminals

Example Context-Free Grammar

grammar

start S

non-terminals E T F

terminals "+" "*" "(" ")" ID

productions

S = E

E = E "+" T

E = T

T = T "*" F

T = F

F = "(" E ")"

F = ID

Abbreviated Grammar

```
grammar
start S
non-terminals E T F
terminals "+" "*" "(" ")" ID
productions
S = E
E = E "+" T
E = T
T = T "*" F
T = F
F = "(" E ")"
F = ID
```

```
grammar
productions
S = E
E = E "+" T
E = T
T = T "*" F
T = F
F = "(" E ")"
F = ID
```

Nonterminals, terminals can be derived from productions

First production defines start symbol

Notation

A, B, C: non-terminals

Σ : terminals

a, b, c: strings of non-terminals and terminals
(alpha, beta, gamma in math)

w, v: strings of terminal symbols

Meta: Syntax of Grammars

```
context-free syntax // grammars
```

```
Grammar.Grammar = <  
  grammar  
    <Start?>  
    <Sorts?>  
    <Terminals?>  
    <Productions>  
>
```

```
context-free syntax
```

```
Production.Prod = <  
  <Symbol><Constructor?> = <Symbol*>  
>
```

```
Symbol.NT = <<ID>>  
Symbol.T  = <<STRING>>  
Symbol.L  = <<LCID>>
```

```
Constructor.Con = <.<ID>>
```

```
context-free syntax
```

```
Start.Start = <  
  start <ID>  
>
```

```
Sorts.Sorts = <  
  sorts <ID*>  
>
```

```
Sorts.NonTerminals = <  
  non-terminals <ID*>  
>
```

```
Terminals.Terminals = <  
  terminals <Symbol*>  
>
```

```
Productions.Productions = <  
  productions  
    <Production*>  
>
```

Derivations: Generating Sentences from Symbols

Derivations

grammar

productions

$E = E \text{ "+" } E$

$E = E \text{ "*" } E$

$E = \text{"-"} E$

$E = \text{"(" } E \text{ ")"}$

$E = \text{ID}$

```
// derivation step: replace symbol by rhs of production
// E = E "+" E
// replace E by E "+" E
//
// derivation:
// repeatedly apply derivations
```

derivation

E

$\Rightarrow \text{"-"} E$

$\Rightarrow \text{"-"} \text{"(" } E \text{ ")"}$

$\Rightarrow \text{"-"} \text{"(" } \text{ID } \text{")"}$

derivation // derives in zero or more steps

$E \Rightarrow^* \text{"-"} \text{"(" } \text{ID } \text{"+" } \text{ID } \text{")"}$

Meta: Syntax of Derivations

```
context-free syntax // derivations
```

```
Derivation.Derivation = <  
  derivation  
  <Symbol> <Step*>  
>
```

```
Step.Step    = [ $\Rightarrow$  [Symbol*]]  
Step.Steps   = [ $\Rightarrow^*$  [Symbol*]]  
Step.Steps1  = [ $\Rightarrow^+$  [Symbol*]]
```

Left-Most Derivation

grammar

productions

$E = E + E$

$E = E * E$

$E = - E$

$E = (E)$

$E = ID$

derivation // left-most derivation

E

$\Rightarrow - E$

$\Rightarrow - (E)$

$\Rightarrow - (E + E)$

$\Rightarrow - (ID + E)$

$\Rightarrow - (ID + ID)$

Left-most derivation: Expand left-most non-terminal at each step

Right-Most Derivation

grammar

productions

$E = E + E$

$E = E * E$

$E = - E$

$E = (E)$

$E = ID$

derivation // left-most derivation

E

$\Rightarrow - E$

$\Rightarrow - (E)$

$\Rightarrow - (E + E)$

$\Rightarrow - (ID + E)$

$\Rightarrow - (ID + ID)$

derivation // right-most derivation

E

$\Rightarrow - E$

$\Rightarrow - (E)$

$\Rightarrow - (E + E)$

$\Rightarrow - (E + ID)$

$\Rightarrow - (ID + ID)$

Right-most derivation: Expand right-most non-terminal at each step

Meta: Tree Derivations

```
context-free syntax // tree derivations

Derivation.TreeDerivation = <
  tree derivation
  <Symbol> <PStep*>
>

PStep.Step    = [⇒ [PT*]]
PStep.Steps   = [⇒* [PT*]]
PStep.Steps1  = [⇒+ [PT*]]

PT.App = <<Symbol> [<PT*>]>
PT.Str = <<STRING>>
PT.Sym = <<Symbol>>
```

Left-Most Tree Derivation

grammar

productions

$E.A = E "+" E$

$E.T = E "*" E$

$E.N = "-" E$

$E.P = "(" E ")"$

$E.V = ID$

derivation // left-most derivation

E

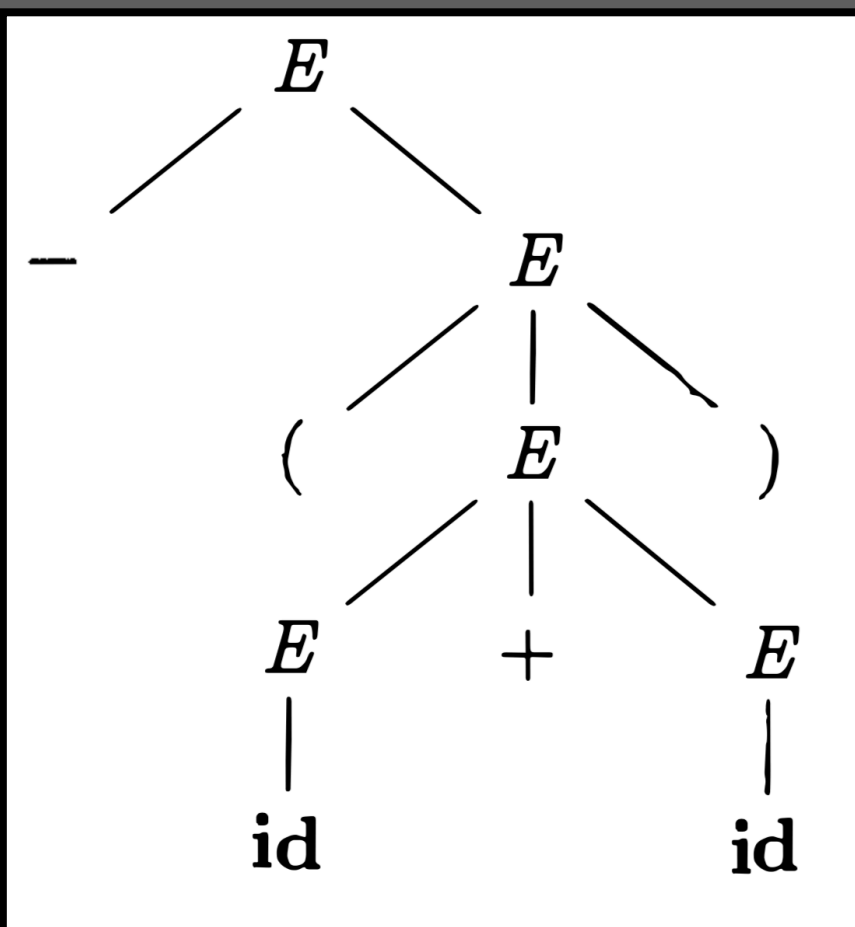
$\Rightarrow "-" E$

$\Rightarrow "-" "(" E ")"$

$\Rightarrow "-" "(" E "+" E ")"$

$\Rightarrow "-" "(" ID "+" E ")"$

$\Rightarrow "-" "(" ID "+" ID ")"$



tree derivation // left-most

E

$\Rightarrow E["-" E]$

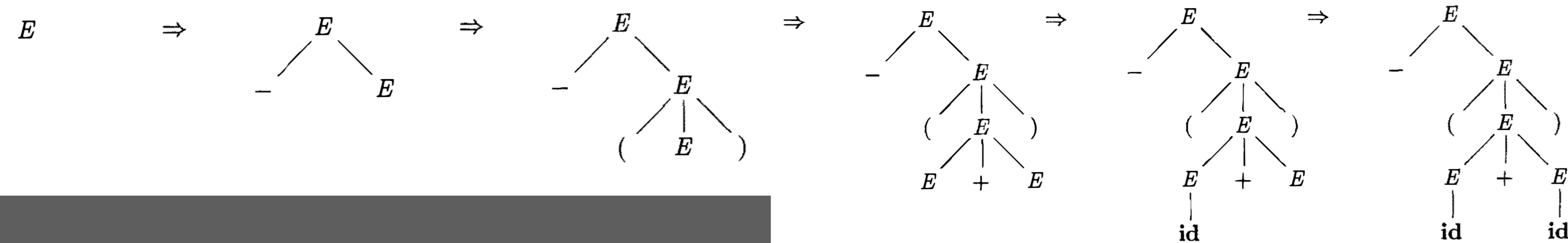
$\Rightarrow E["-" E["(" E ")"]]$

$\Rightarrow E["-" E["(" E[E "+" E] ")"]]$

$\Rightarrow E["-" E["(" E[E[ID] "+" E] ")"]]$

$\Rightarrow E["-" E["(" E[E[ID] "+" E[ID]] ")"]]$

Left-Most Tree Derivation



```
tree derivation // left-most
```

```
E
```

```
 $\Rightarrow E["-"] E$ 
```

```
 $\Rightarrow E["-"] E["(" E ")"]$ 
```

```
 $\Rightarrow E["-"] E["(" E[E "+"] E ")"]$ 
```

```
 $\Rightarrow E["-"] E["(" E[E[ID] "+"] E ")"]$ 
```

```
 $\Rightarrow E["-"] E["(" E[E[ID] "+"] E[ID] ")"]$ 
```

Meta: Term Derivations

```
context-free syntax // term derivations

Derivation.TermDerivation = <
  term derivation
  <Symbol> <TStep*>
>

TStep.Step    = [⇒ [Term*]]
TStep.Steps   = [⇒* [Term*]]
TStep.Steps1  = [⇒+ [Term*]]

Term.App = <<ID> (<{Term " , " }*>)>
Term.Str = <<STRING>>
Term.Sym = <<Symbol>>
```

Left-Most Term Derivation

grammar

productions

$E.A = E "+" E$

$E.T = E "*" E$

$E.N = "-" E$

$E.P = "(" E ")"$

$E.V = ID$

derivation // left-most derivation

E

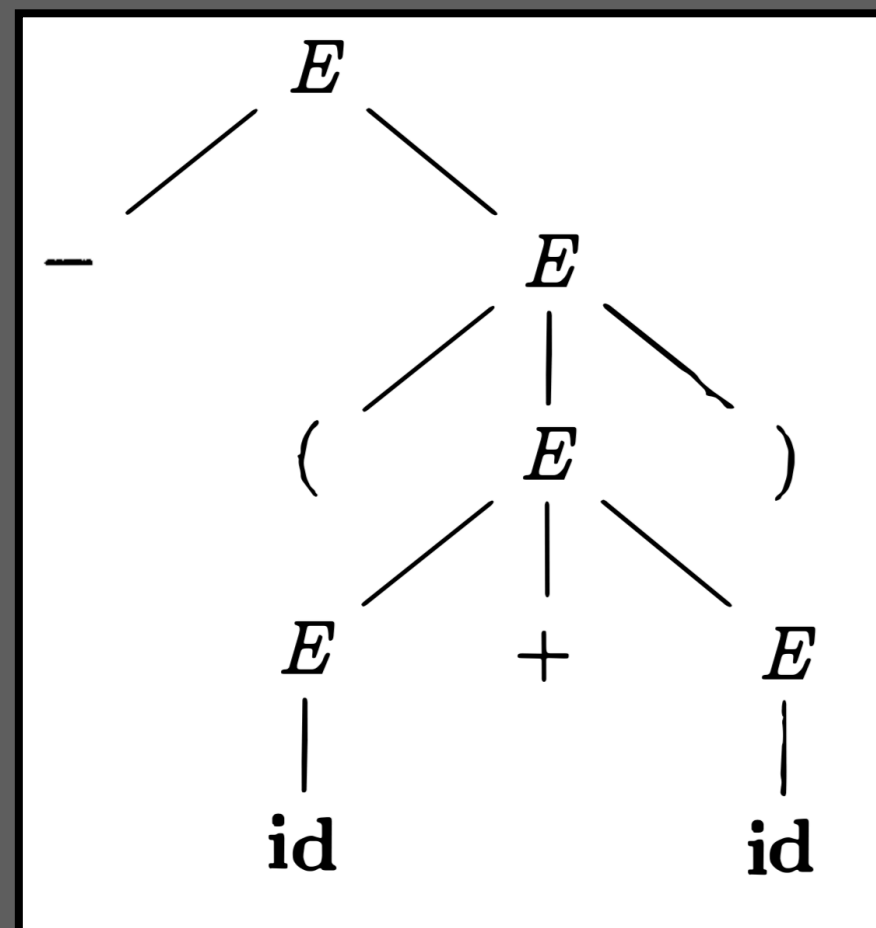
$\Rightarrow "-" E$

$\Rightarrow "-" "(" E ")"$

$\Rightarrow "-" "(" E "+" E ")"$

$\Rightarrow "-" "(" ID "+" E ")"$

$\Rightarrow "-" "(" ID "+" ID ")"$



term derivation // left-most

E

$\Rightarrow N(E)$

$\Rightarrow N(P(E))$

$\Rightarrow N(P(A(E, E)))$

$\Rightarrow N(P(A(V(ID), E)))$

$\Rightarrow N(P(A(V(ID), V(ID))))$

Parse Trees Represent Derivations

```
List<String> YIELD(T : Tree) {  
  T match {  
    [A = Ts] => YIELDS(Ts);  
    Str => [Str];  
  };  
}  
  
List<String> YIELDS(Ts : List<Tree>) {  
  Ts match {  
    [] => "";  
    [T | Ts] => YIELD(T) ++ YIELDS(Ts);  
  };  
}
```

$$S \Rightarrow^* PT$$

iff

$$S \Rightarrow^* \text{YIELD}(PT)$$

Language Defined by a Grammar

$$L(G) = \{ w \mid S \Rightarrow^* w \}$$

Language: sentences

$$T(G) = \{ T \mid S \Rightarrow^* T \}$$

Language: trees

$$L(G) = \text{YIELD}(T(G))$$

Ambiguity

Ambiguity: Deriving Multiple Parse Trees

grammar

productions

$E.A = E \text{ "+" } E$

$E.T = E \text{ "*" } E$

$E.N = \text{"-" } E$

$E.P = \text{"(" } E \text{ ")"}$

$E.V = \text{ID}$

derivation

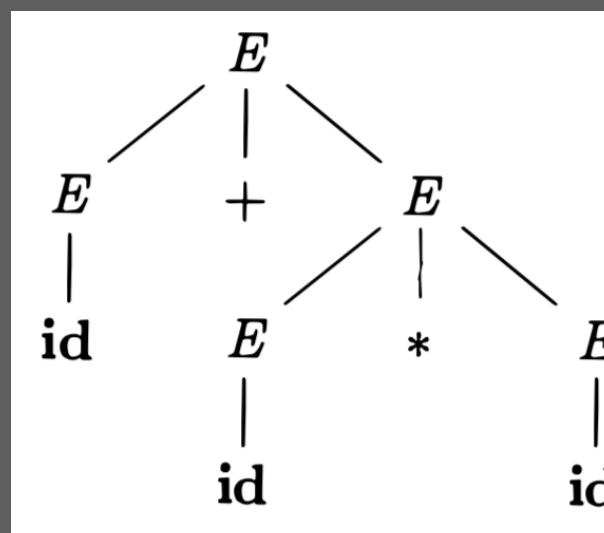
$E \Rightarrow^* \text{ID "+" ID "*" ID}$

derivation

E
 $\Rightarrow E \text{ "+" } E$
 $\Rightarrow \text{ID "+" } E$
 $\Rightarrow \text{ID "+" } E \text{ "*" } E$
 $\Rightarrow \text{ID "+" ID "*" } E$
 $\Rightarrow \text{ID "+" ID "*" ID}$

tree derivation

$E \Rightarrow^* E[E[\text{ID}] \text{ "+" } E[E[\text{ID}] \text{ "*" } E[\text{ID}]]]$

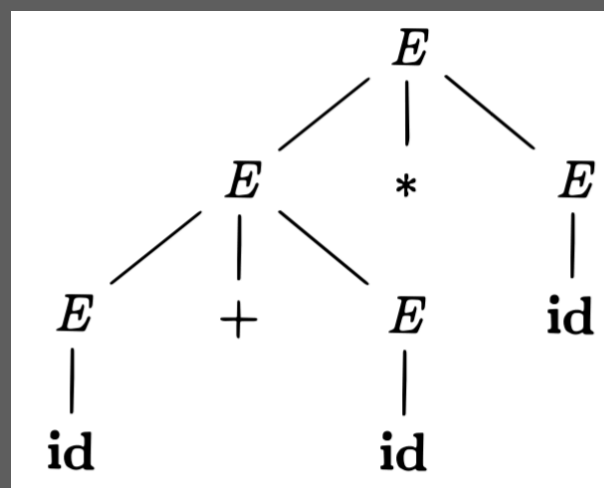


derivation

E
 $\Rightarrow E \text{ "*" } E$
 $\Rightarrow E \text{ "+" } E \text{ "*" } E$
 $\Rightarrow \text{ID "+" } E \text{ "*" } E$
 $\Rightarrow \text{ID "+" ID "*" } E$
 $\Rightarrow \text{ID "+" ID "*" ID}$

tree derivation

$E \Rightarrow^* E[E[E[\text{ID}] \text{ "+" } E[\text{ID}]] \text{ "*" } E[\text{ID}]]$



Ambiguous grammar: produces >1 parse tree for a sentence

Ambiguity: Deriving Abstract Syntax Terms

grammar

productions

$E.A = E \text{ "+" } E$
 $E.T = E \text{ "*" } E$
 $E.N = \text{"-"} E$
 $E.P = \text{"(" } E \text{ ")"}$
 $E.V = \text{ID}$

derivation

$E \Rightarrow * \text{ ID "+" ID "*" ID}$

derivation

E
 $\Rightarrow E \text{ "+" } E$
 $\Rightarrow \text{ID "+" } E$
 $\Rightarrow \text{ID "+" } E \text{ "*" } E$
 $\Rightarrow \text{ID "+" ID "*" } E$
 $\Rightarrow \text{ID "+" ID "*" ID}$

term derivation

E
 $\Rightarrow A(E, E)$
 $\Rightarrow A(V(\text{ID}), E)$
 $\Rightarrow A(V(\text{ID}), T(E, E))$
 $\Rightarrow A(V(\text{ID}), T(V(\text{ID}), E))$
 $\Rightarrow A(V(\text{ID}), T(V(\text{ID}), V(\text{ID})))$

derivation

E
 $\Rightarrow E \text{ "*" } E$
 $\Rightarrow E \text{ "+" } E \text{ "*" } E$
 $\Rightarrow \text{ID "+" } E \text{ "*" } E$
 $\Rightarrow \text{ID "+" ID "*" } E$
 $\Rightarrow \text{ID "+" ID "*" ID}$

term derivation

E
 $\Rightarrow T(E, E)$
 $\Rightarrow T(A(E, E), E)$
 $\Rightarrow T(A(V(\text{ID}), E), E)$
 $\Rightarrow T(A(V(\text{ID}), V(\text{ID})), E)$
 $\Rightarrow T(A(V(\text{ID}), V(\text{ID})), V(\text{ID}))$

Disambiguation

Traditional: Ambiguity = Parse Table Conflict

context-free syntax

```
Exp          = <(<Exp>)> {bracket}

Exp.Int      = INT
Exp.Var      = ID
Exp.Add      = <<Exp> + <Exp>>

Exp.Fun      = <function(<{ID " ,"}*>) <Exp>>
Exp.App      = <<Exp> <Exp>>

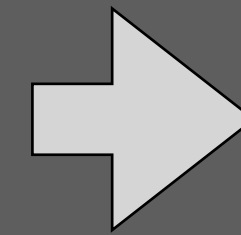
Exp.Let      = <let <Bnd*> in <Exp>>

Bnd.Bnd      = <<ID> = <Exp>>

Exp.If       = <if(<Exp>) <Exp>>
Exp.IfElse   = <if(<Exp>) <Exp> else <Exp>>

Exp.Match    = <match <Exp> with <{Case "|" }+>>
Case.Case    = [[Pat] → [Exp]]

Pat.PVar     = ID
Pat.PApp     = <<Pat> <Pat>>
```



No can parse

Ambiguity = Multiple Possible Parses

context-free syntax

```
Exp          = <(<Exp>)> {bracket}

Exp.Int      = INT
Exp.Var      = ID
Exp.Add      = <<Exp> + <Exp>>

Exp.Fun      = <function(<{ID " ,"}*>) <Exp>>
Exp.App      = <<Exp> <Exp>>

Exp.Let      = <let <Bnd*> in <Exp>>

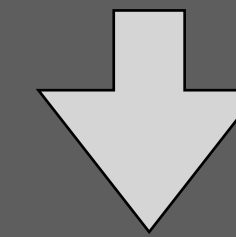
Bnd.Bnd      = <<ID> = <Exp>>

Exp.If       = <if(<Exp>) <Exp>>
Exp.IfElse   = <if(<Exp>) <Exp> else <Exp>>

Exp.Match    = <match <Exp> with <{Case "|" }+>>
Case.Case    = [[Pat] → [Exp]]

Pat.PVar     = ID
Pat.PApp     = <<Pat> <Pat>>
```

a + b + c



```
amb(
  [ Add(Var("a"), Add(Var("b"), Var("c")))
    , Add(Add(Var("a"), Var("b")), Var("c"))
  ]
)
```

Disambiguation = Select(Structure)

context-free syntax

```
Exp          = <(<Exp>)> {bracket}

Exp.Int      = INT
Exp.Var      = ID
Exp.Add      = <<Exp> + <Exp>>

Exp.Fun      = <function(<{ID " ,"}*>) <Exp>>
Exp.App      = <<Exp> <Exp>>

Exp.Let      = <let <Bnd*> in <Exp>>

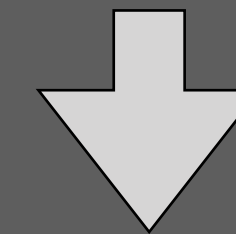
Bnd.Bnd      = <<ID> = <Exp>>

Exp.If       = <if(<Exp>) <Exp>>
Exp.IfElse   = <if(<Exp>) <Exp> else <Exp>>

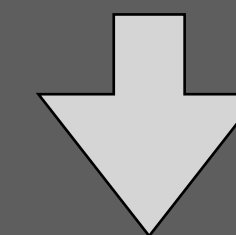
Exp.Match    = <match <Exp> with <{Case "|" }+>>
Case.Case    = [[Pat] → [Exp]]

Pat.PVar     = ID
Pat.PApp     = <<Pat> <Pat>>
```

a + b + c



```
amb(
  [ Add(Var("a"), Add(Var("b"), Var("c")))
    , Add(Add(Var("a"), Var("b")), Var("c"))
  ]
)
```



Add(Add(Var("a"), Var("b")), Var("c"))

Brackets = Explicit Disambiguation

context-free syntax

```
Exp          = <(<Exp>)> {bracket}

Exp.Int      = INT
Exp.Var      = ID
Exp.Add      = <<Exp> + <Exp>>

Exp.Fun      = <function(<{ID " ,"}*>) <Exp>>
Exp.App      = <<Exp> <Exp>>

Exp.Let      = <let <Bnd*> in <Exp>>

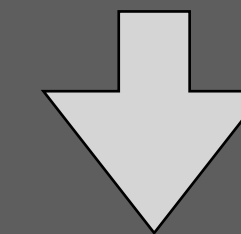
Bnd.Bnd      = <<ID> = <Exp>>

Exp.If       = <if(<Exp>) <Exp>>
Exp.IfElse   = <if(<Exp>) <Exp> else <Exp>>

Exp.Match    = <match <Exp> with <{Case "|" }+>>
Case.Case    = [[Pat] → [Exp]]

Pat.PVar     = ID
Pat.PApp     = <<Pat> <Pat>>
```

a + (b + c)



Add(Var("a"), Add(Var("b"), Var("c")))

Disambiguation by Manual Transformation = Bad

context-free syntax

```
Exp          = <(<Exp>)> {bracket}

Exp.Int      = INT
Exp.Var      = ID
Exp.Add      = <<Exp> + <Exp>>

Exp.Fun      = <function(<{ID " ,"}*>) <Exp>>
Exp.App      = <<Exp> <Exp>>

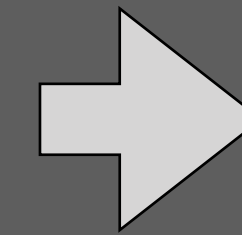
Exp.Let      = <let <Bnd*> in <Exp>>

Bnd.Bnd      = <<ID> = <Exp>>

Exp.If       = <if(<Exp>) <Exp>>
Exp.IfElse   = <if(<Exp>) <Exp> else <Exp>>

Exp.Match    = <match <Exp> with <{Case "|" }+>>
Case.Case    = [[Pat] → [Exp]]

Pat.PVar     = ID
Pat.PApp     = <<Pat> <Pat>>
```



Big ugly grammar

Declarative Disambiguation = Separate Concern

context-free syntax

```
Exp          = <(<Exp>)> {bracket}

Exp.Int      = INT
Exp.Var      = ID
Exp.Add      = <<Exp> + <Exp>> {left}

Exp.Fun      = <function(<{ID " , "}*>) <Exp>>
Exp.App      = <<Exp> <Exp>> {left}

Exp.Let      = <let <Bnd*> in <Exp>>

Bnd.Bnd      = <<ID> = <Exp>>

Exp.If       = <if(<Exp>) <Exp>>
Exp.IfElse   = <if(<Exp>) <Exp> else <Exp>>

Exp.Match    = <match <Exp> with <{Case "|" }*>>
               {longest-match}
Case.Case    = [[Pat] → [Exp]]

Pat.PVar     = ID
Pat.PApp     = <<Pat> <Pat>> {left}
```

context-free priorities

```
Exp.App > Exp.Add > Exp.IfElse > Exp.If
> Exp.Match > Exp.Let > Exp.Fun
```

Associativity = Solve Intra Operator Ambiguity

context-free syntax

Exp = <(<Exp>)> {**bracket**}

Exp.Int = INT

Exp.Var = ID

Exp.Add = <<Exp> + <Exp>> {**left**}

Exp.Fun = <function(<{ID " , "}*>) <Exp>>

Exp.App = <<Exp> <Exp>> {**left**}

Exp.Let = <let <Bnd*> in <Exp>>

Bnd.Bnd = <<ID> = <Exp>>

Exp.If = <if(<Exp>) <Exp>>

Exp.IfElse = <if(<Exp>) <Exp> else <Exp>>

Exp.Match = <match <Exp> with <{Case "|" }*>>
{**longest-match**}

Case.Case = [[Pat] → [Exp]]

Pat.PVar = ID

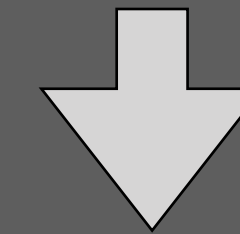
Pat.PApp = <<Pat> <Pat>> {**left**}

context-free priorities

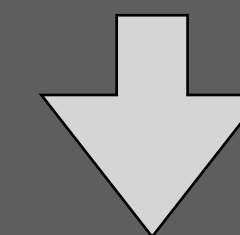
Exp.App > Exp.Add > Exp.IfElse > Exp.If

> Exp.Match > Exp.Let > Exp.Fun

a + b + c



```
amb(  
  [ Add(Var("a"), Add(Var("b"), Var("c")))  
    , Add(Add(Var("a"), Var("b")), Var("c"))  
  ]  
)
```



Add(Add(Var("a"), Var("b")), Var("c"))

Priority = Solve Inter Operator Ambiguity

context-free syntax

Exp = <(<Exp>)> {**bracket**}

Exp.Int = INT

Exp.Var = ID

Exp.Add = <<Exp> + <Exp>> {**left**}

Exp.Fun = <function(<{ID " , "}*>) <Exp>>

Exp.App = <<Exp> <Exp>> {**left**}

Exp.Let = <let <Bnd*> in <Exp>>

Bnd.Bnd = <<ID> = <Exp>>

Exp.If = <if(<Exp>) <Exp>>

Exp.IfElse = <if(<Exp>) <Exp> else <Exp>>

Exp.Match = <match <Exp> with <{Case "|" }*>>
{**longest-match**}

Case.Case = [[Pat] → [Exp]]

Pat.PVar = ID

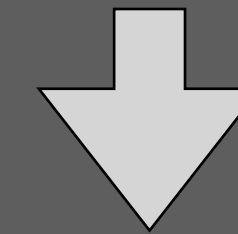
Pat.PApp = <<Pat> <Pat>> {**left**}

context-free priorities

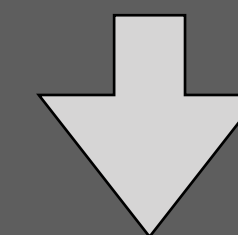
Exp.App > Exp.Add > Exp.IfElse > Exp.If

> Exp.Match > Exp.Let > Exp.Fun

f a + b



```
amb(  
  [ Add(App(Var("f"), Var("a")), Var("b"))  
    , App(Var("f"), Add(Var("a"), Var("b")))  
  ]  
)
```



Add(App(Var("f"), Var("a")), Var("b"))

Dangling Else = Operators with Overlapping Prefix

context-free syntax

```
Exp          = <(<Exp>)> {bracket}

Exp.Int      = INT
Exp.Var      = ID
Exp.Add      = <<Exp> + <Exp>> {left}

Exp.Fun      = <function(<{ID " , "}*>) <Exp>>
Exp.App      = <<Exp> <Exp>> {left}

Exp.Let      = <let <Bnd*> in <Exp>>

Bnd.Bnd      = <<ID> = <Exp>>

Exp.If       = <if(<Exp>) <Exp>>
Exp.IfElse   = <if(<Exp>) <Exp> else <Exp>>

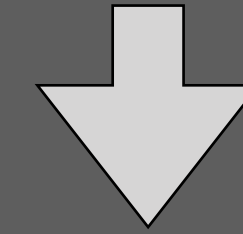
Exp.Match    = <match <Exp> with <{Case "|" }*>>
               {longest-match}
Case.Case    = [[Pat] → [Exp]]
```

```
Pat.PVar     = ID
Pat.PApp     = <<Pat> <Pat>> {left}
```

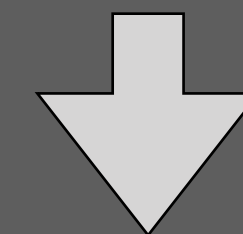
context-free priorities

```
Exp.App > Exp.Add > Exp.IfElse > Exp.If
> Exp.Match > Exp.Let > Exp.Fun
```

```
if(1) if(2) 3 else 4
```



```
amb(
  [ IfElse(
      Int("1")
      , If(Int("2"), Int("3"))
      , Int("4")
    )
  , If(
      Int("1")
      , IfElse(Int("2"), Int("3"), Int("4"))
    )
  ]
)
```



```
If(
  Int("1")
  , IfElse(Int("2"), Int("3"), Int("4"))
)
```

Safe Disambiguation = Do Not Reject Unambiguous Sentences

context-free syntax

```
Exp          = <(<Exp>)> {bracket}

Exp.Int      = INT
Exp.Var      = ID
Exp.Add      = <<Exp> + <Exp>> {left}

Exp.Fun      = <function(<{ID " , "}*>) <Exp>>
Exp.App      = <<Exp> <Exp>> {left}

Exp.Let      = <let <Bnd*> in <Exp>>

Bnd.Bnd      = <<ID> = <Exp>>

Exp.If       = <if(<Exp>) <Exp>>
Exp.IfElse  = <if(<Exp>) <Exp> else <Exp>>

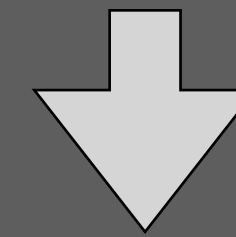
Exp.Match   = <match <Exp> with <{Case "|" }*>>
              {longest-match}
Case.Case   = [[Pat] → [Exp]]

Pat.PVar    = ID
Pat.PApp    = <<Pat> <Pat>> {left}
```

context-free priorities

```
Exp.App > Exp.Add > Exp.IfElse > Exp.If
> Exp.Match > Exp.Let > Exp.Fun
```

4 + if(y) x



Add(Int("4"), If(Var("y"), Var("x")))

Deep Priority Conflict

context-free syntax

Exp = <(<Exp>)> {**bracket**}

Exp.Int = INT

Exp.Var = ID

Exp.Add = <<Exp> + <Exp>> {**left**}

Exp.Fun = <function(<{ID " , "}*>) <Exp>>

Exp.App = <<Exp> <Exp>> {**left**}

Exp.Let = <let <Bnd*> in <Exp>>

Bnd.Bnd = <<ID> = <Exp>>

Exp.If = <if(<Exp>) <Exp>>

Exp.IfElse = <if(<Exp>) <Exp> else <Exp>>

Exp.Match = <match <Exp> with <{Case "|" }*>>
{**longest-match**}

Case.Case = [[Pat] → [Exp]]

Pat.PVar = ID

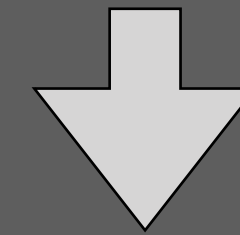
Pat.PApp = <<Pat> <Pat>> {**left**}

context-free priorities

Exp.App > Exp.Add > Exp.IfElse > Exp.If

> Exp.Match > Exp.Let > Exp.Fun

4 + if(y) x + 3



```
amb(  
  [ Add(  
    Int("4")  
    , amb(  
      [ Add(If(Var("y"), Var("x")), Int("3"))  
        , If(Var("y"), Add(Var("x"), Int("3")))  
      ]  
    )  
  )  
  , Add(  
    Add(Int("4"), If(Var("y"), Var("x")))  
    , Int("3")  
  )  
]  
)
```

Deep Priority Conflict (Solved)

context-free syntax

Exp = <(<Exp>)> {**bracket**}

Exp.Int = INT

Exp.Var = ID

Exp.Add = <<Exp> + <Exp>> {**left**}

Exp.Fun = <function(<{ID " , "}*>) <Exp>>

Exp.App = <<Exp> <Exp>> {**left**}

Exp.Let = <let <Bnd*> in <Exp>>

Bnd.Bnd = <<ID> = <Exp>>

Exp.If = <if(<Exp>) <Exp>>

Exp.IfElse = <if(<Exp>) <Exp> else <Exp>>

Exp.Match = <match <Exp> with <{Case "|" }*>>
{**longest-match**}

Case.Case = [[Pat] → [Exp]]

Pat.PVar = ID

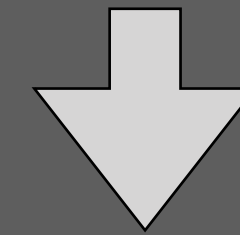
Pat.PApp = <<Pat> <Pat>> {**left**}

context-free priorities

Exp.App > Exp.Add > Exp.IfElse > Exp.If

> Exp.Match > Exp.Let > Exp.Fun

4 + if(y) x + 3



```
Add(  
  Int("4")  
  , If(Var("y"), Add(Var("x"), Int("3")))  
)
```

Longest Match = Solve Repetition Ambiguity

context-free syntax

```
Exp          = <(<Exp>)> {bracket}

Exp.Int      = INT
Exp.Var      = ID
Exp.Add      = <<Exp> + <Exp>> {left}

Exp.Fun      = <function(<{ID " , "}*>) <Exp>>
Exp.App      = <<Exp> <Exp>> {left}

Exp.Let      = <let <Bnd*> in <Exp>>

Bnd.Bnd      = <<ID> = <Exp>>

Exp.If       = <if(<Exp>) <Exp>>
Exp.IfElse   = <if(<Exp>) <Exp> else <Exp>>

Exp.Match    = <match <Exp> with <{Case "|" }*>>
               {longest-match}
Case.Case    = [[Pat] → [Exp]]

Pat.PVar     = ID
Pat.PApp     = <<Pat> <Pat>> {left}
```

context-free priorities

```
Exp.App > Exp.Add > Exp.IfElse > Exp.If
> Exp.Match > Exp.Let > Exp.Fun
```

```
match x with
  a → match 5 with
      b → 3
      | c → 4
```

```
Match(
  Var("x")
, amb(
  [ [ Case(
      PVar("a")
      , Match(
          Int("5")
          , [Case(PVar("b"), Int("3"))]
        )
      )
    , Case(PVar("c"), Int("4"))
  ]
, [ Case(
      PVar("a")
      , Match(
          Int("5")
          , [ Case(PVar("b"), Int("3"))
            , Case(PVar("c"), Int("4"))
          ]
        )
      )
  ]
)
)
)
```


Longest Match = Solve Repetition Ambiguity

context-free syntax

```
Exp          = <(<Exp>)> {bracket}

Exp.Int      = INT
Exp.Var      = ID
Exp.Add      = <<Exp> + <Exp>> {left}

Exp.Fun      = <function(<{ID " , "}*>) <Exp>>
Exp.App      = <<Exp> <Exp>> {left}

Exp.Let      = <let <Bnd*> in <Exp>>

Bnd.Bnd      = <<ID> = <Exp>>

Exp.If       = <if(<Exp>) <Exp>>
Exp.IfElse  = <if(<Exp>) <Exp> else <Exp>>

Exp.Match    = <match <Exp> with <{Case "|" }*>>
               {longest-match}

Case.Case    = [[Pat] → [Exp]]

Pat.PVar     = ID
Pat.PApp     = <<Pat> <Pat>> {left}
```

context-free priorities

```
Exp.App > Exp.Add > Exp.IfElse > Exp.If
> Exp.Match > Exp.Let > Exp.Fun
```

```
match x with
  a → match 5 with
      b → 3
      | c → 4
```

```
Match(
  Var("x")
, [ Case(
    PVar("a")
    , Match(
      Int("5")
      , [ Case(PVar("b"), Int("3"))
          , Case(PVar("c"), Int("4"))
        ]
    )
  )
]
)
```

Parenthesize

Parenthesize = Disambiguate⁻¹ (Insert Necessary Parentheses)

context-free syntax

```
Exp          = <(<Exp>)> {bracket}

Exp.Int      = INT
Exp.Var      = ID
Exp.Add      = <<Exp> + <Exp>> {left}

Exp.Fun      = <function(<{ID " , "}*>) <Exp>>
Exp.App      = <<Exp> <Exp>> {left}

Exp.Let      = <let <Bnd*> in <Exp>>

Bnd.Bnd      = <<ID> = <Exp>>

Exp.If       = <if(<Exp>) <Exp>>
Exp.IfElse   = <if(<Exp>) <Exp> else <Exp>>

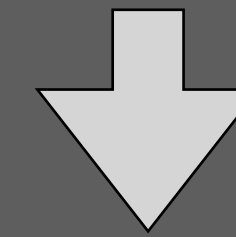
Exp.Match    = <match <Exp> with <{Case "|" }*>>
               {longest-match}
Case.Case    = [[Pat] → [Exp]]

Pat.PVar     = ID
Pat.PApp     = <<Pat> <Pat>> {left}
```

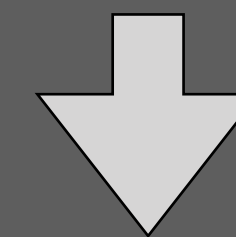
context-free priorities

```
Exp.App > Exp.Add > Exp.IfElse > Exp.If
> Exp.Match > Exp.Let > Exp.Fun
```

(a + b) + c



Add(Add(Var("a"), Var("b")), Var("c"))



a + b + c

Parenthesize = Disambiguate⁻¹ (Insert Necessary Parentheses)

context-free syntax

```
Exp          = <(<Exp>)> {bracket}

Exp.Int      = INT
Exp.Var      = ID
Exp.Add      = <<Exp> + <Exp>> {left}

Exp.Fun      = <function(<{ID " , "}*>) <Exp>>
Exp.App      = <<Exp> <Exp>> {left}

Exp.Let      = <let <Bnd*> in <Exp>>

Bnd.Bnd      = <<ID> = <Exp>>

Exp.If       = <if(<Exp>) <Exp>>
Exp.IfElse   = <if(<Exp>) <Exp> else <Exp>>

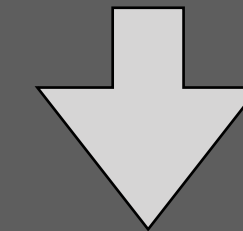
Exp.Match    = <match <Exp> with <{Case "|" }*>>
               {longest-match}
Case.Case    = [[Pat] → [Exp]]

Pat.PVar     = ID
Pat.PApp     = <<Pat> <Pat>> {left}
```

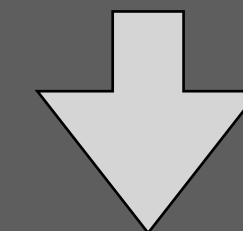
context-free priorities

```
Exp.App > Exp.Add > Exp.IfElse > Exp.If
> Exp.Match > Exp.Let > Exp.Fun
```

```
a + (let x = b in (c + d))
```



```
Add(
  Var("a")
, Let(
  [Bnd("x", Var("b"))]
, Add(Var("c"), Var("d"))
)
)
```



```
a + let
  x = b
in
  c + d
```

Parenthesize = Disambiguate⁻¹ (Insert Necessary Parentheses)

context-free syntax

Exp = <(<Exp>)> {**bracket**}

Exp.Int = INT

Exp.Var = ID

Exp.Add = <<Exp> + <Exp>> {**left**}

Exp.Fun = <function(<{ID " , "}*>) <Exp>>

Exp.App = <<Exp> <Exp>> {**left**}

Exp.Let = <let <Bnd*> in <Exp>>

Bnd.Bnd = <<ID> = <Exp>>

Exp.If = <if(<Exp>) <Exp>>

Exp.IfElse = <if(<Exp>) <Exp> else <Exp>>

Exp.Match = <match <Exp> with <{Case " | "}*>>
{**longest-match**}

Case.Case = [[Pat] → [Exp]]

Pat.PVar = ID

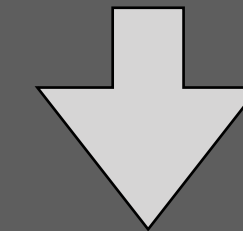
Pat.PApp = <<Pat> <Pat>> {**left**}

context-free priorities

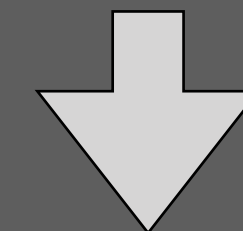
Exp.App > Exp.Add > Exp.IfElse > Exp.If

> Exp.Match > Exp.Let > Exp.Fun

(a + (let x = b in c)) + d



```
Add(
  Add(
    Var("a")
    , Let([Bnd("x", Var("b"))], Var("c"))
  )
  , Var("d")
)
```



a + (let
x = b
in
c) + d

Parenthesize = Disambiguate⁻¹ (Insert Necessary Parentheses)

context-free syntax

Exp = <(<Exp>)> {**bracket**}

Exp.Int = INT

Exp.Var = ID

Exp.Add = <<Exp> + <Exp>> {**left**}

Exp.Sub = <<Exp> - <Exp>> {**left**}

Exp.Fun = <function(<{ID " , "}*>) <Exp>>

Exp.App = <<Exp> <Exp>> {**left**}

Exp.Let = <let <Bnd*> in <Exp>>

Bnd.Bnd = <<ID> = <Exp>>

Exp.If = <if(<Exp>) <Exp>>

Exp.IfElse = <if(<Exp>) <Exp> else <Exp>>

Exp.Match = <match <Exp> with <{Case " | "}*>>
{**longest-match**}

Case.Case = [[Pat] → [Exp]]

Pat.PVar = ID

Pat.PApp = <<Pat> <Pat>> {**left**}

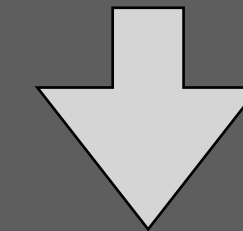
context-free priorities

{**left**: Exp.Sub Exp.App} > Exp.Add > Exp.IfElse >

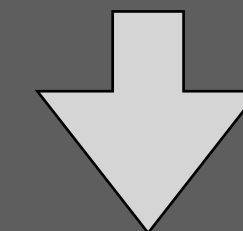
Exp.If

> Exp.Match > Exp.Let > Exp.Fun

```
(a + (let x = b in c)) + d
```



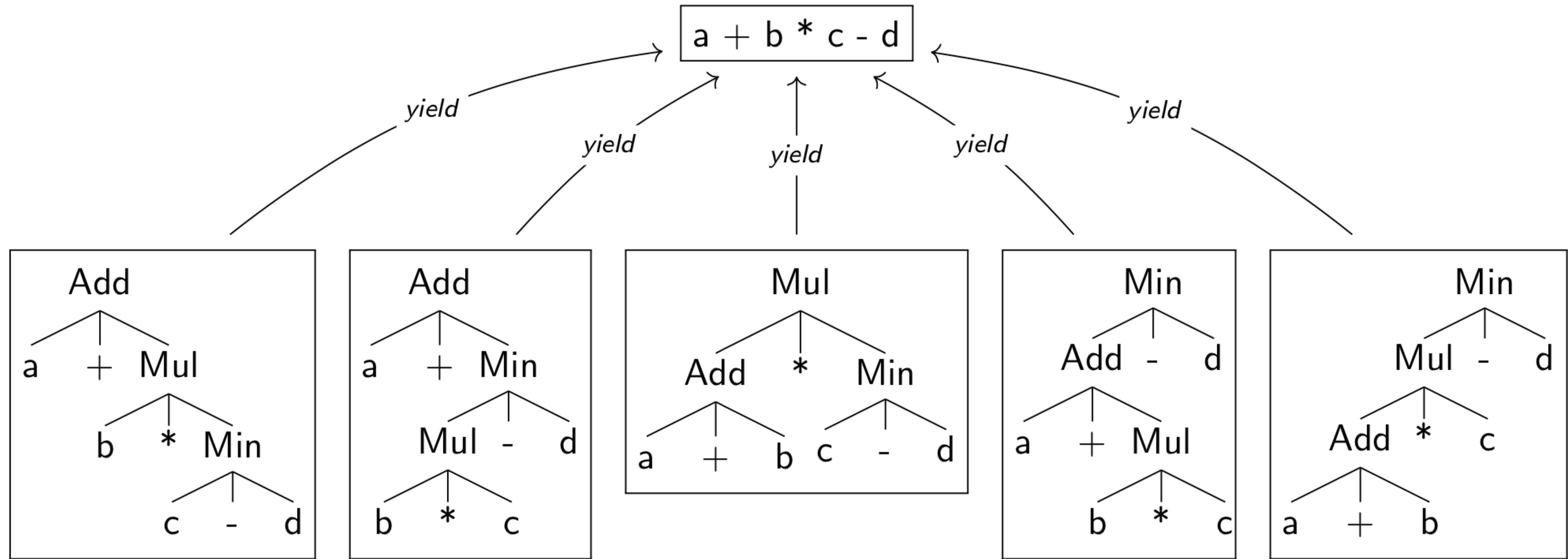
```
Add(  
  Add(  
    Var("a")  
    , Let([Bnd("x", Var("b"))], Var("c"))  
  )  
  , Var("d")  
)
```



```
a + (let  
  x = b  
in  
  c) + d
```

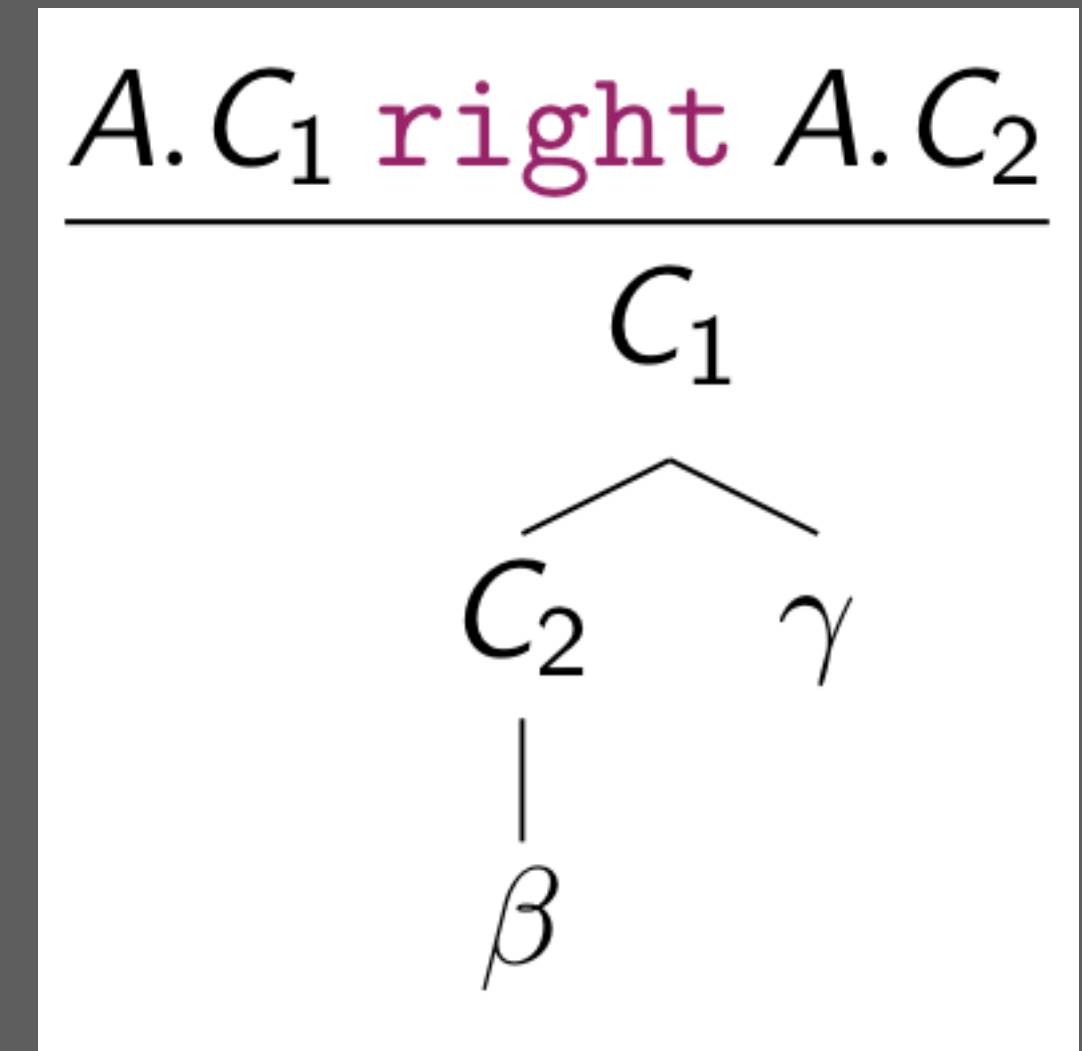
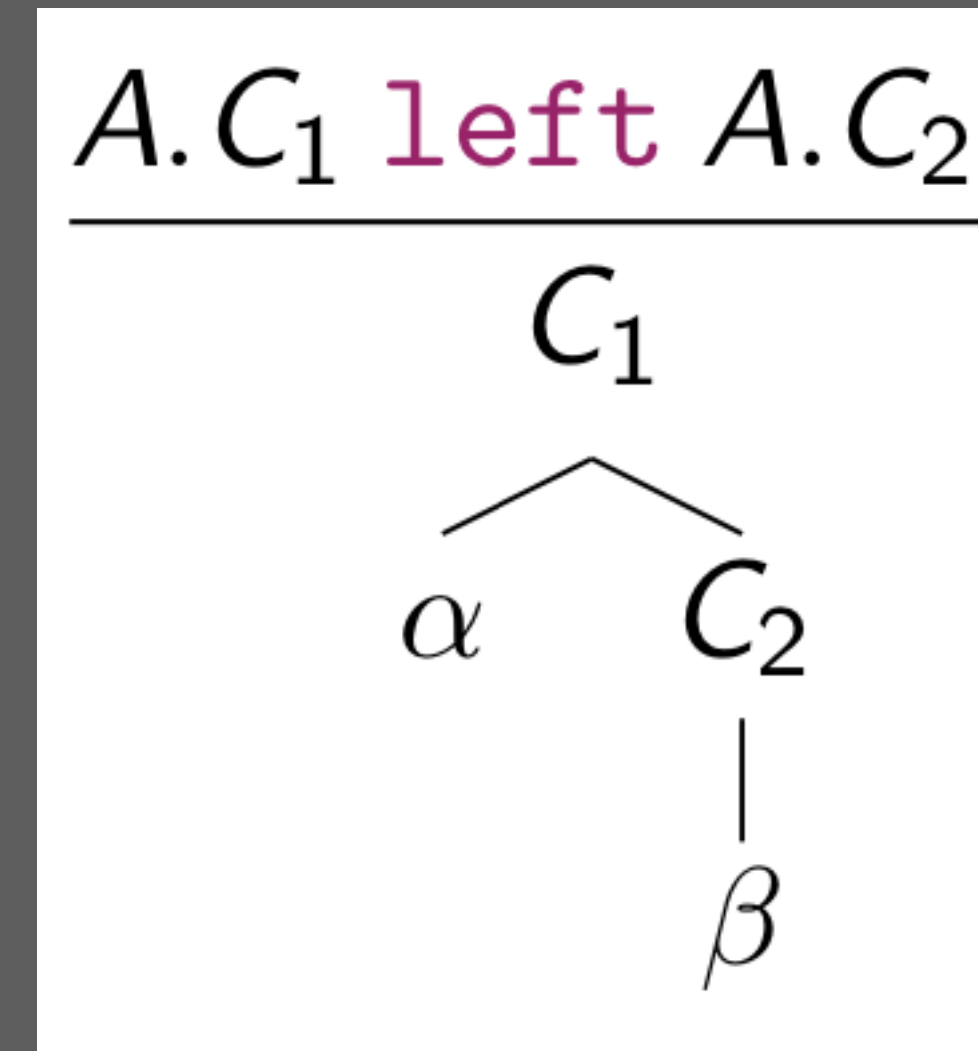
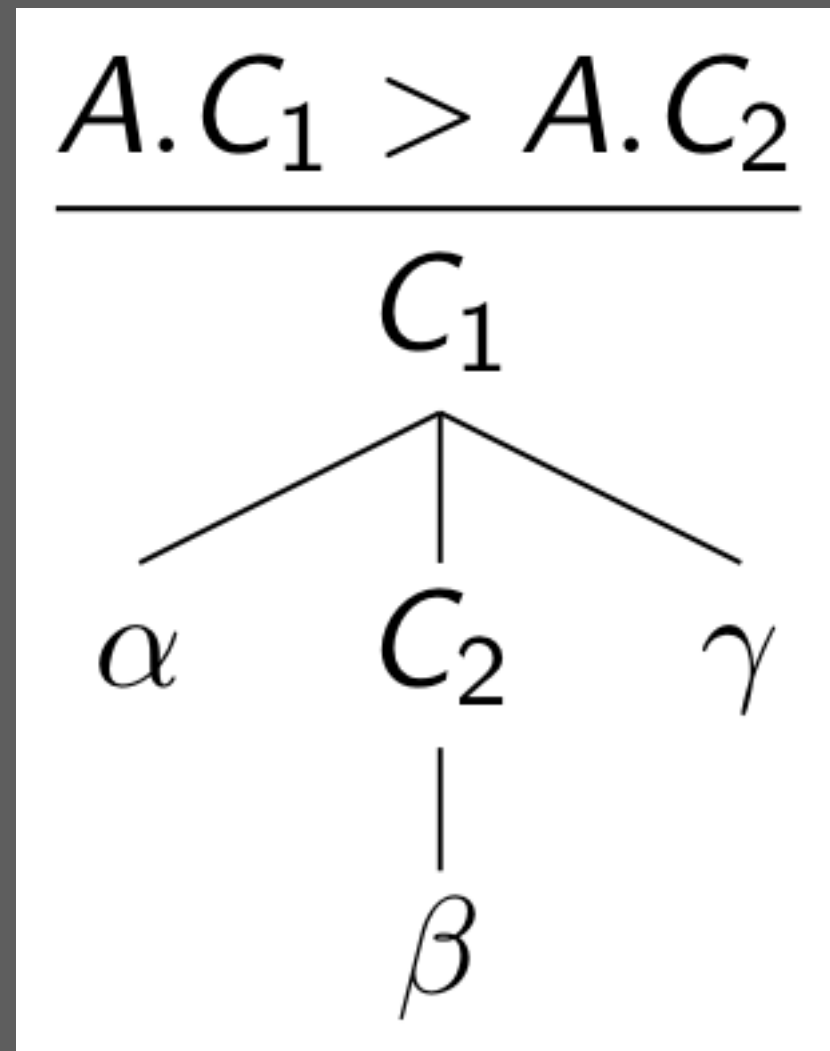
Semantics of Associativity and Priority

Ambiguous Sentence has Multiple Parse Trees



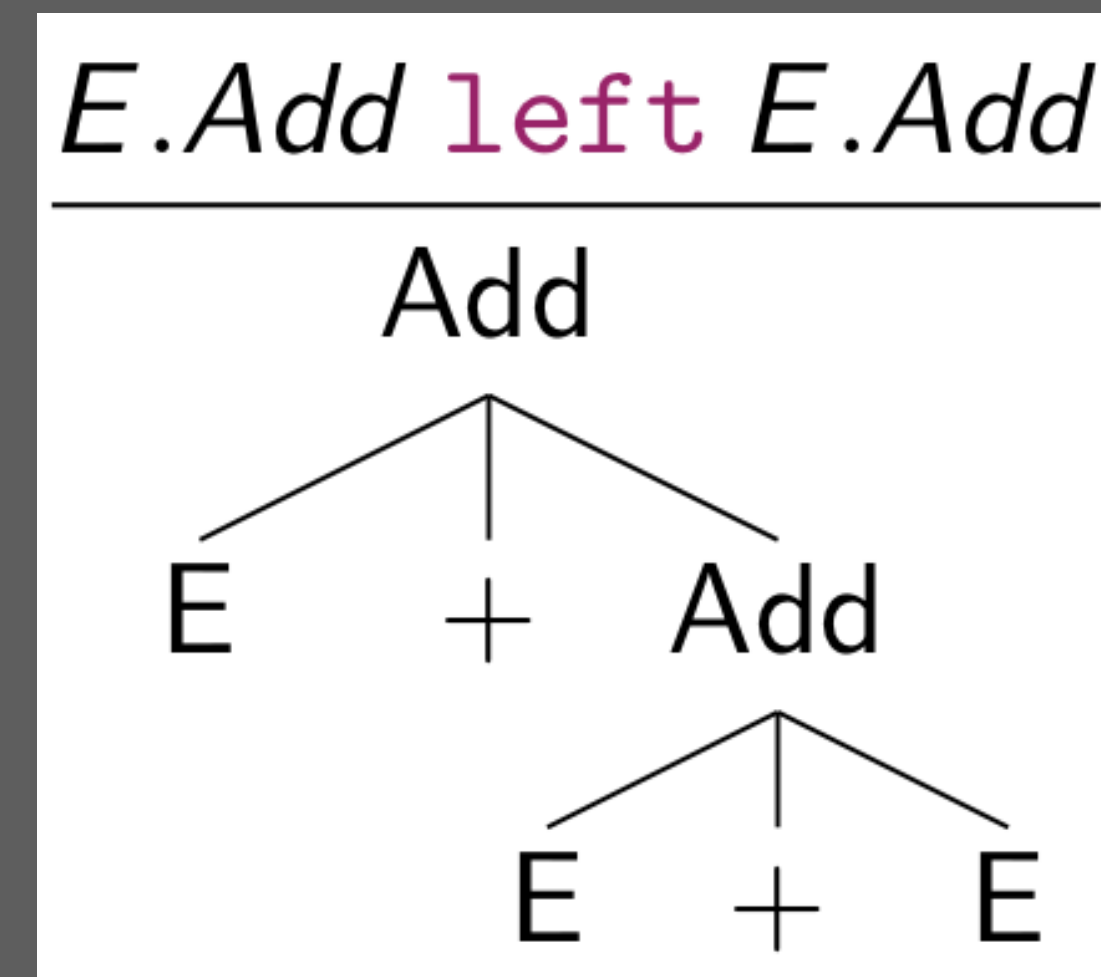
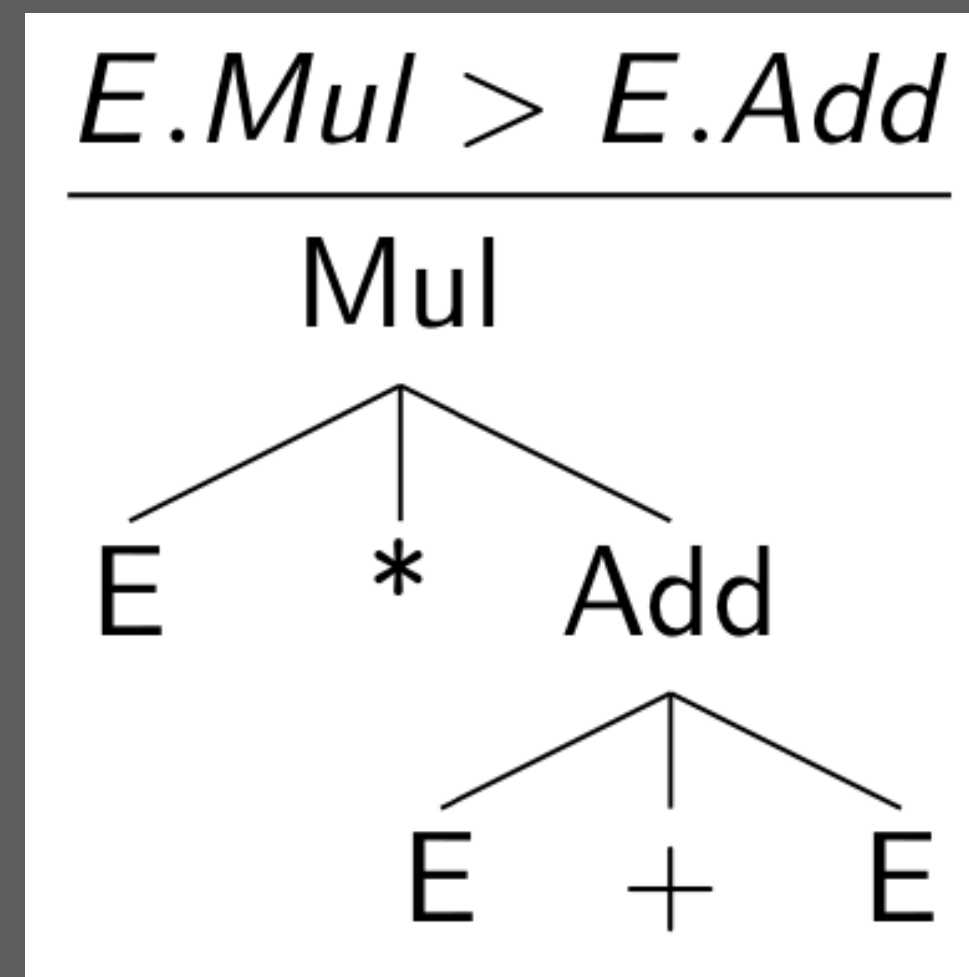
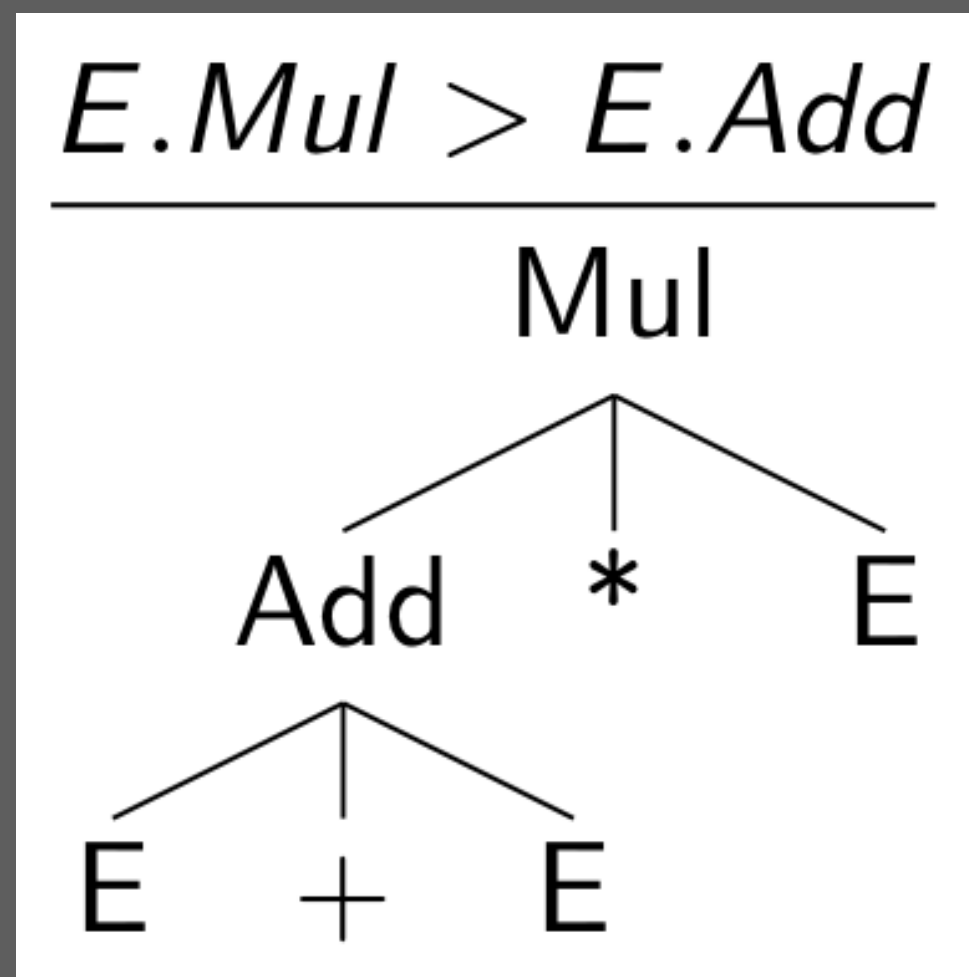
Associativity and Priority as Subtree Exclusion Rules [SDF2 (1997)]

Rules

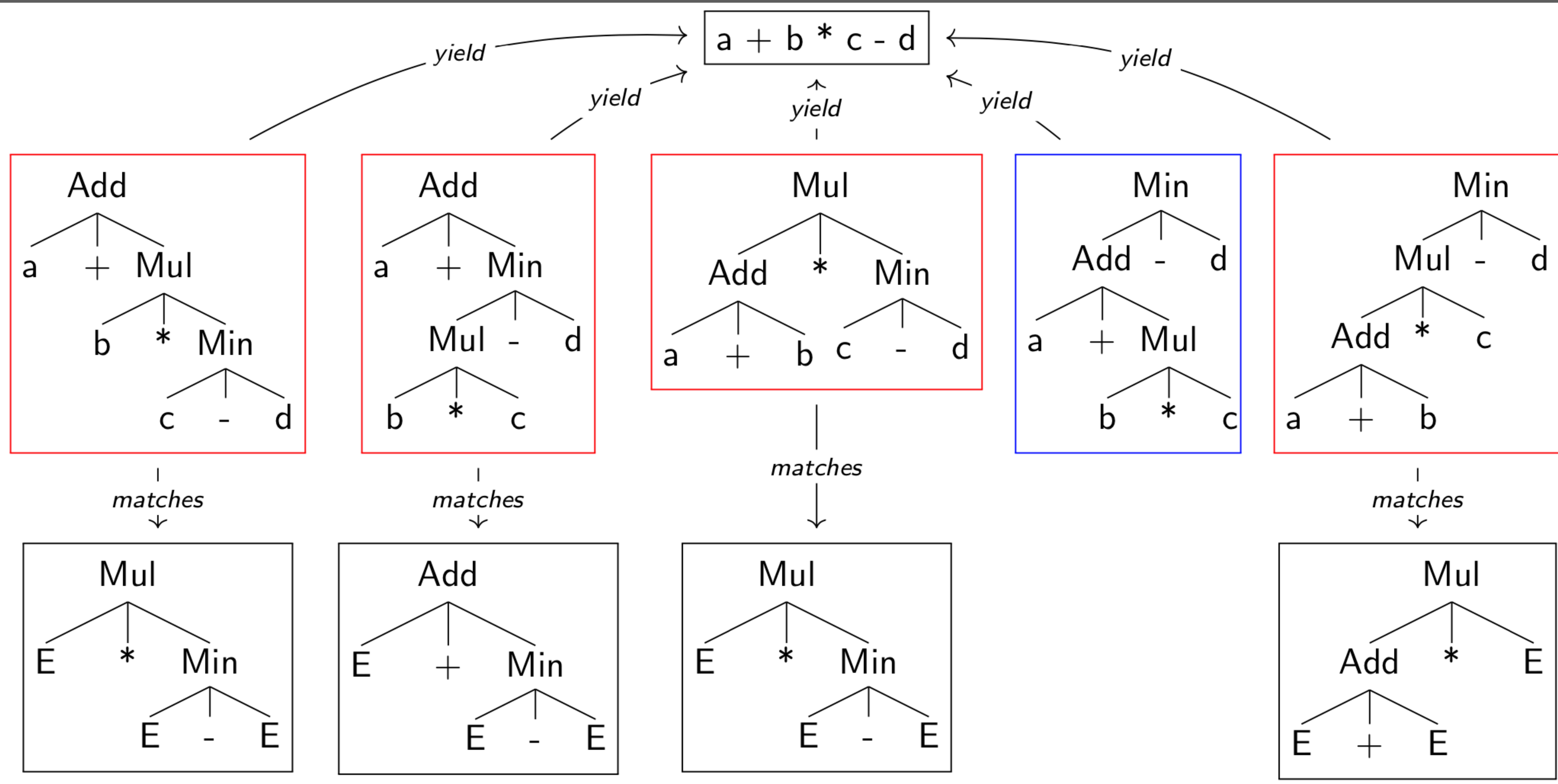


Disambiguation rules generate subtree exclusion patterns (aka conflict patterns)

Instances

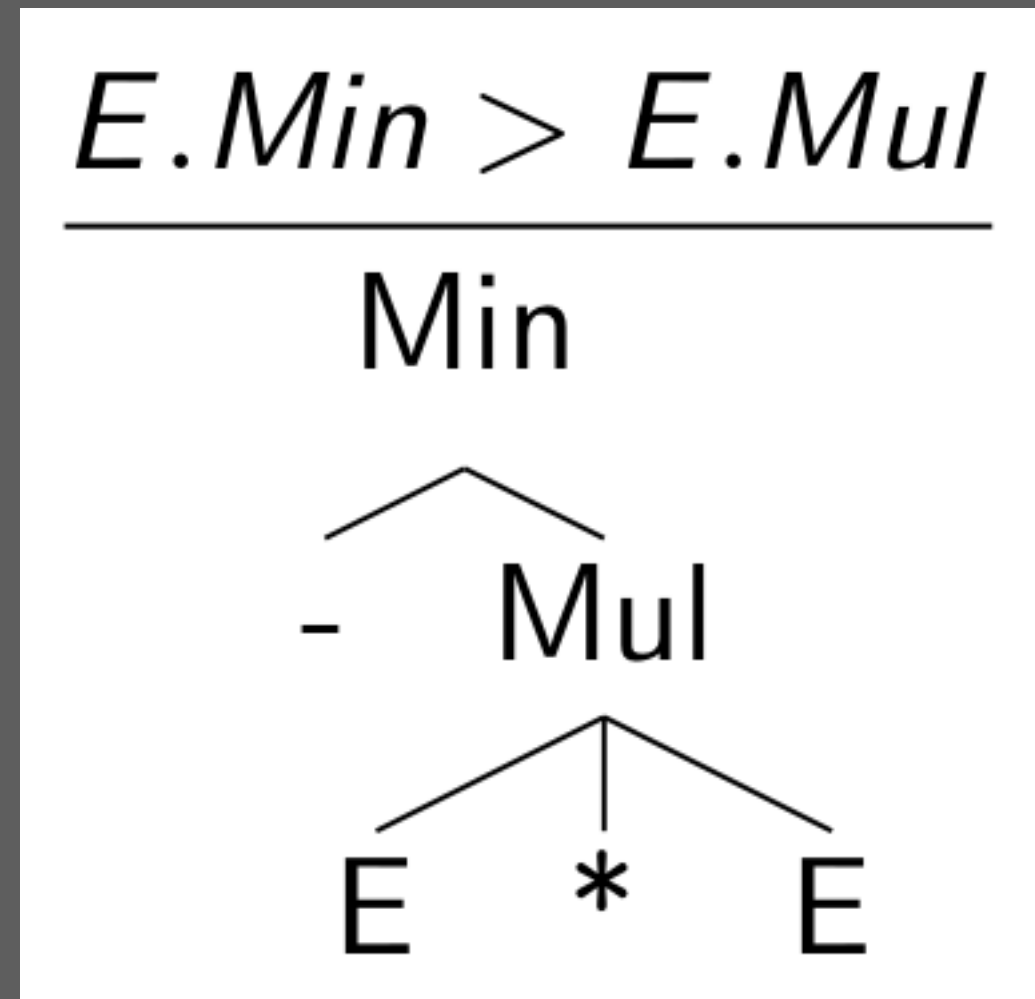
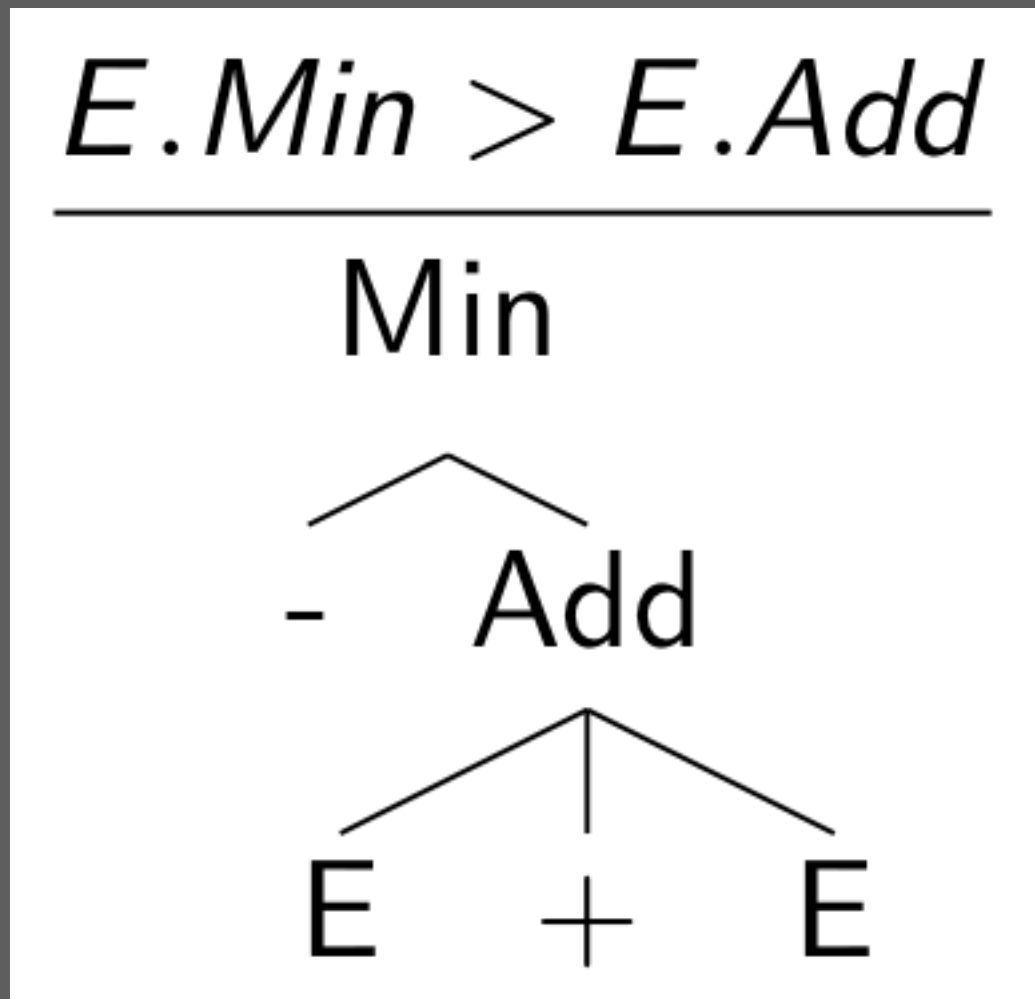


Disambiguation by Subtree Exclusion

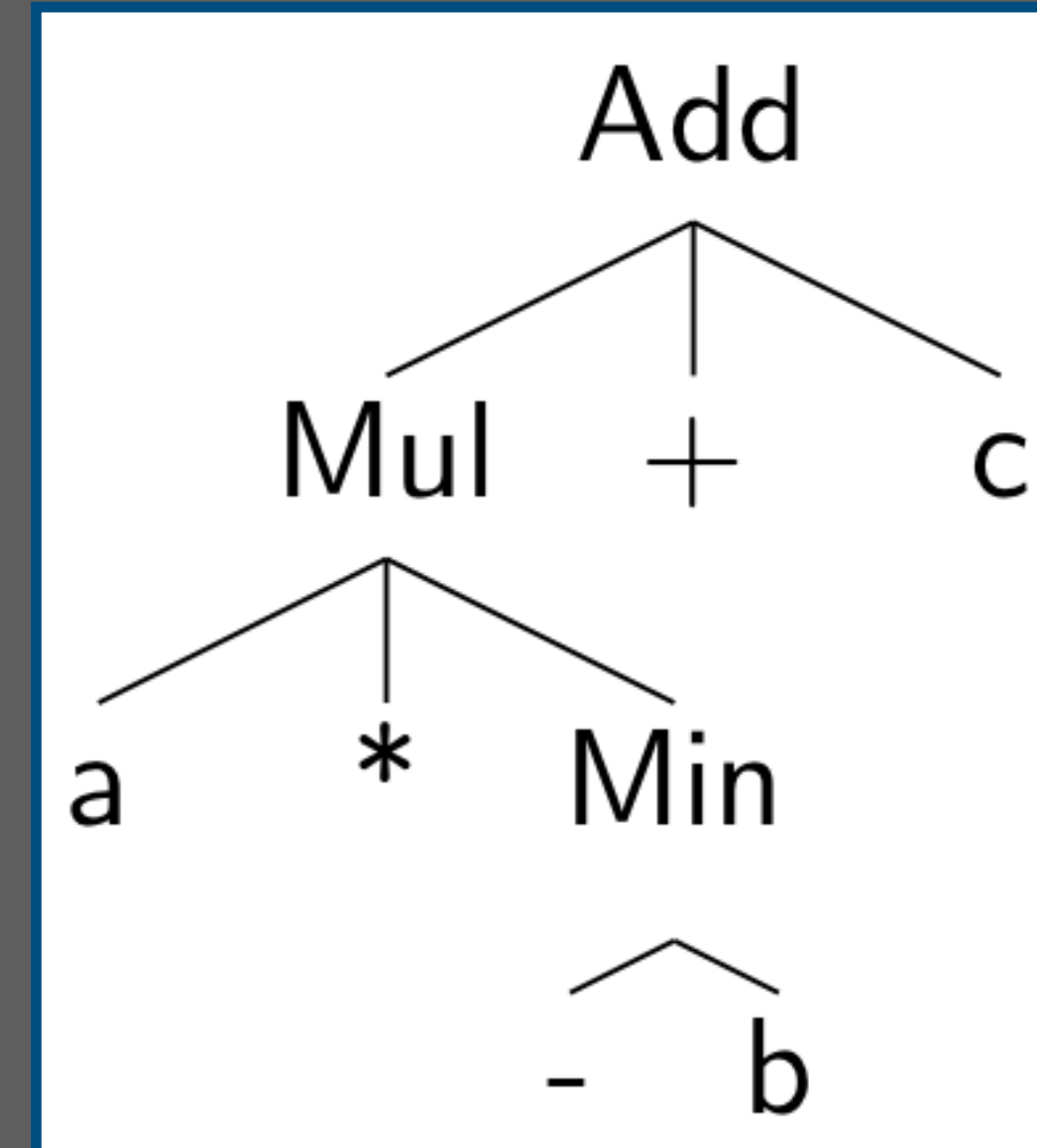
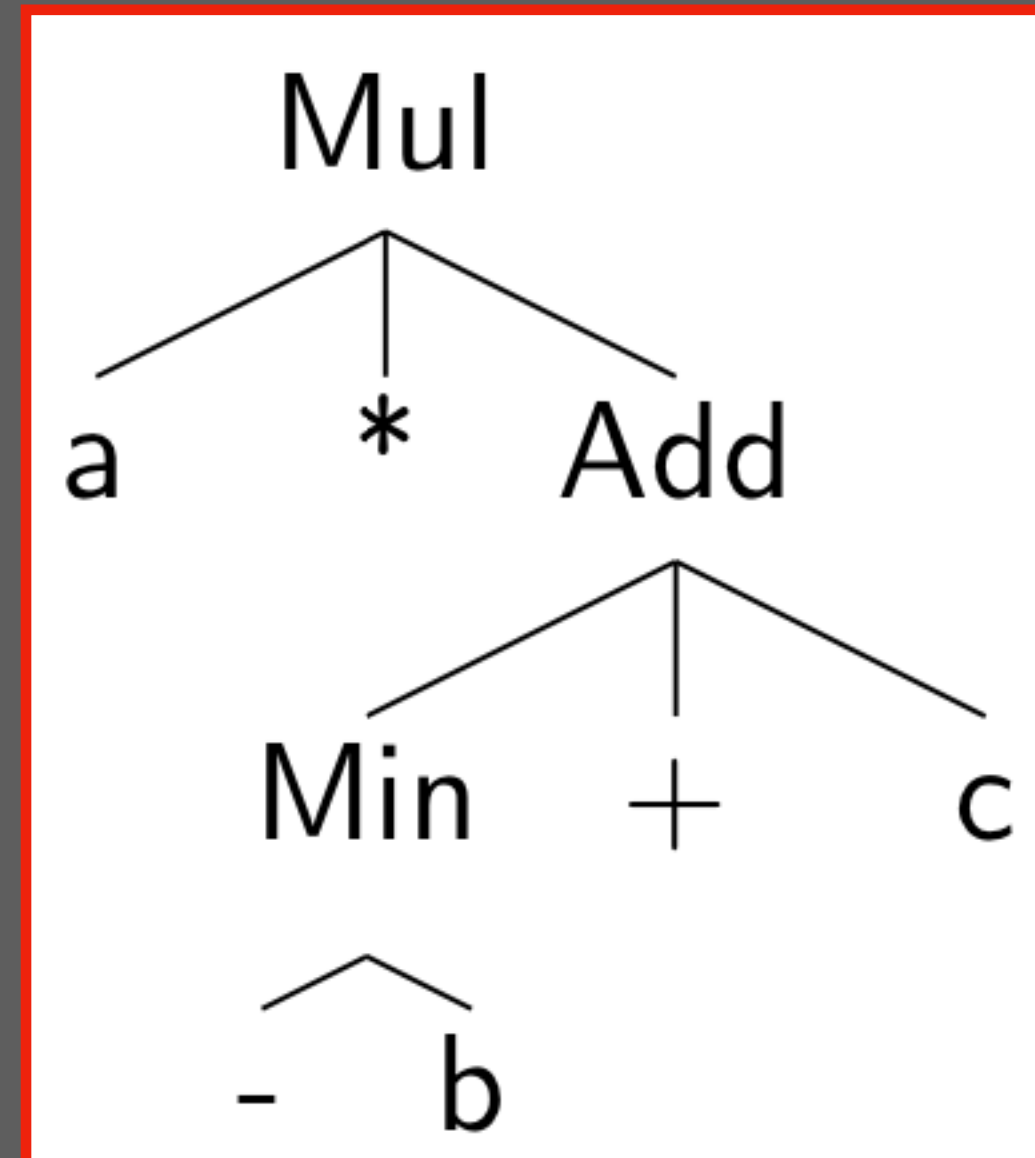
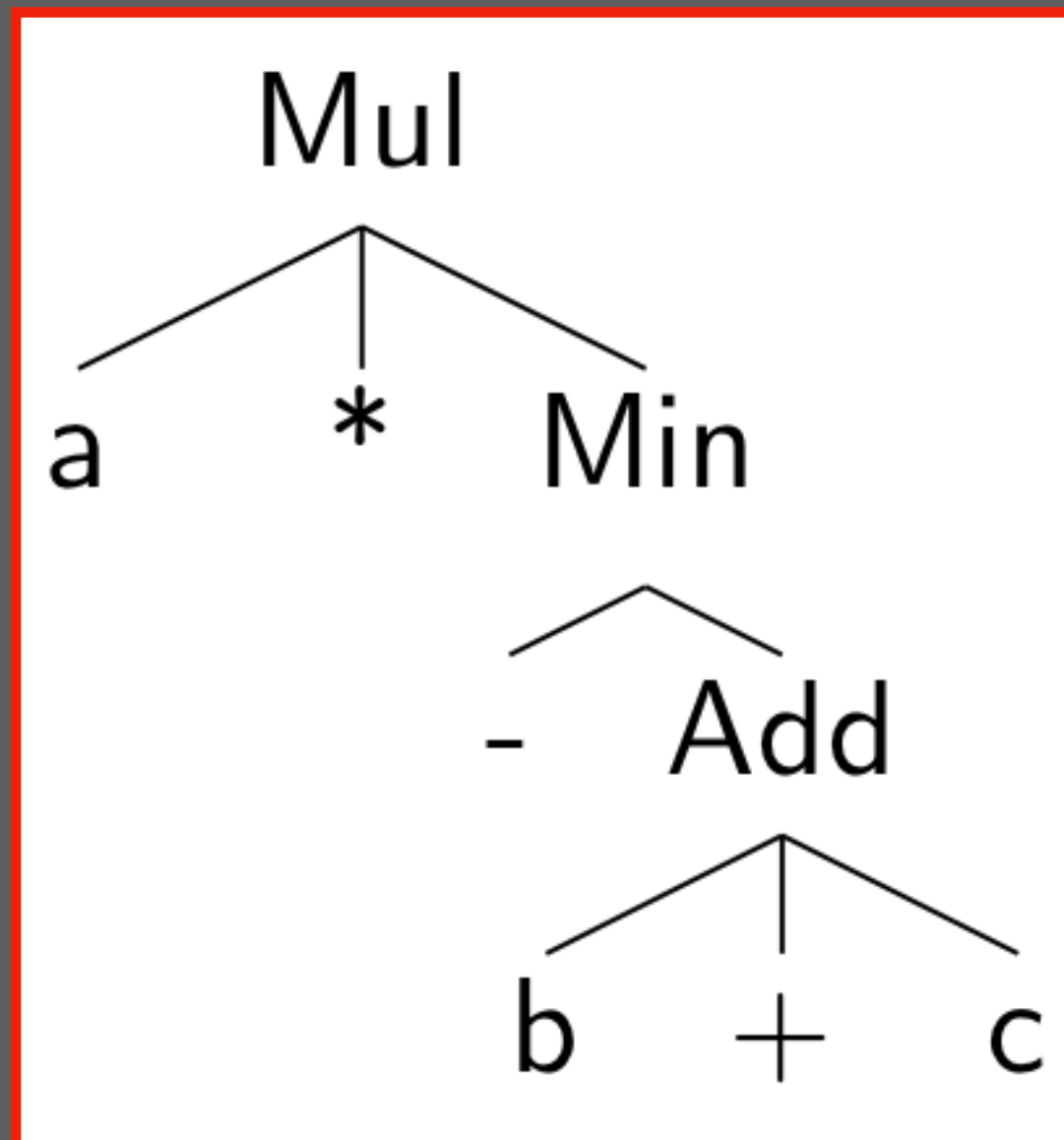


Safe for High Priority Prefix Operators

Conflict
Patterns

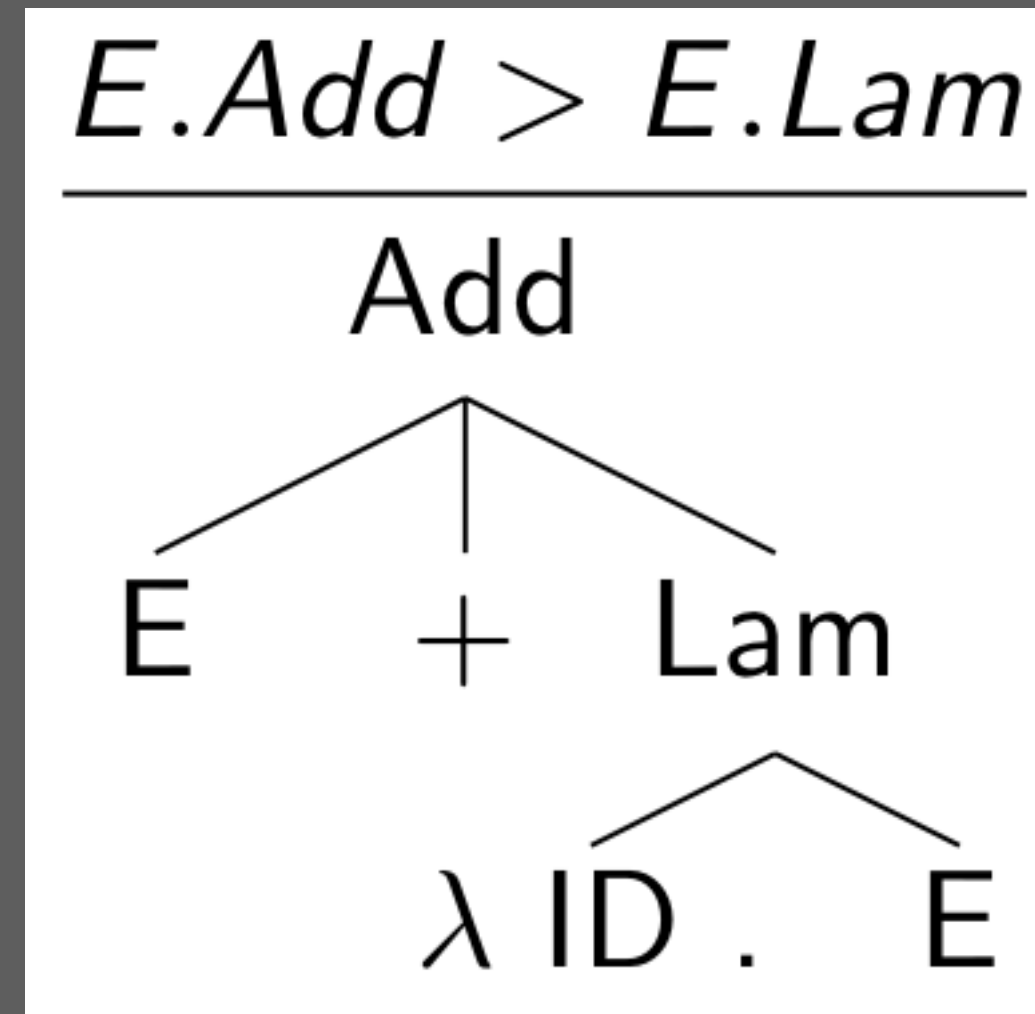
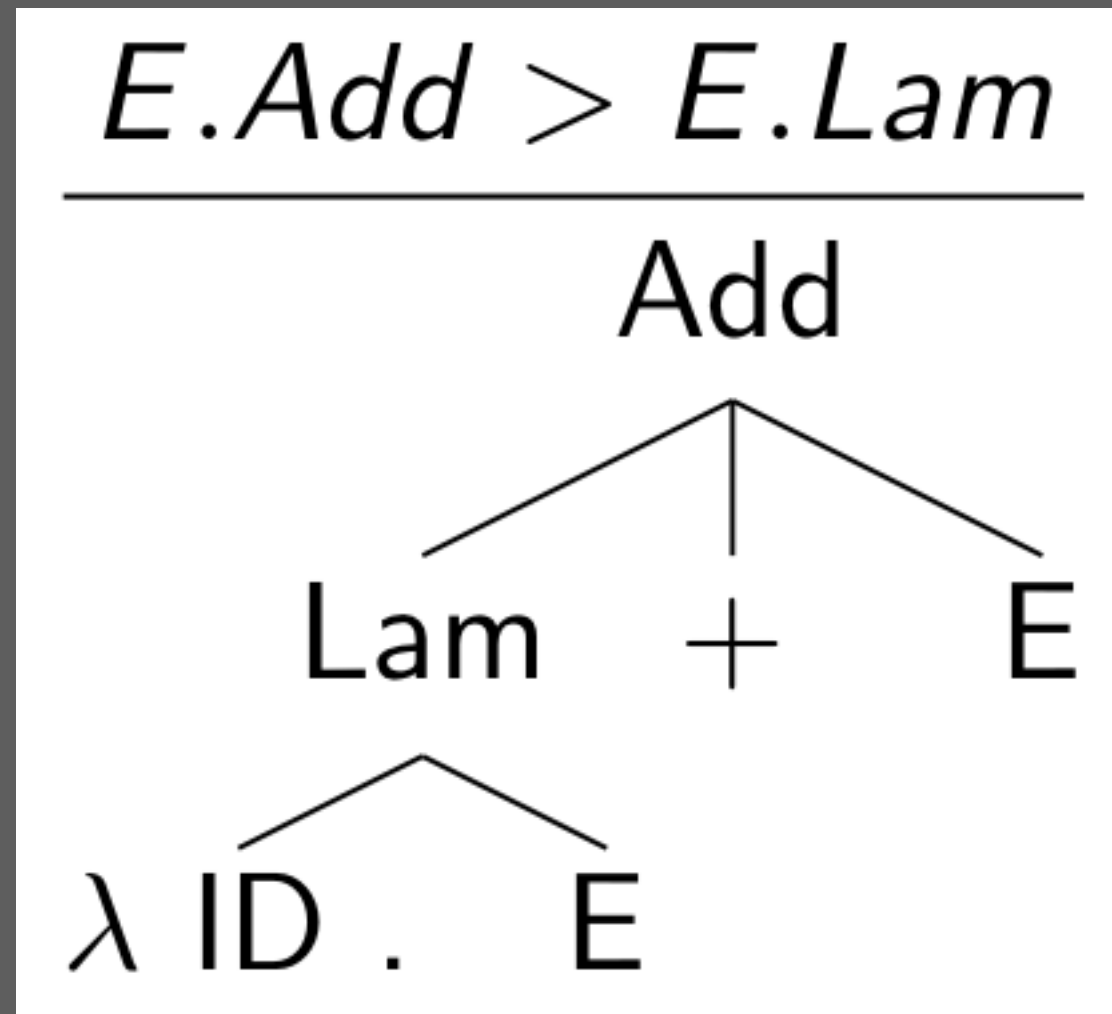


Trees

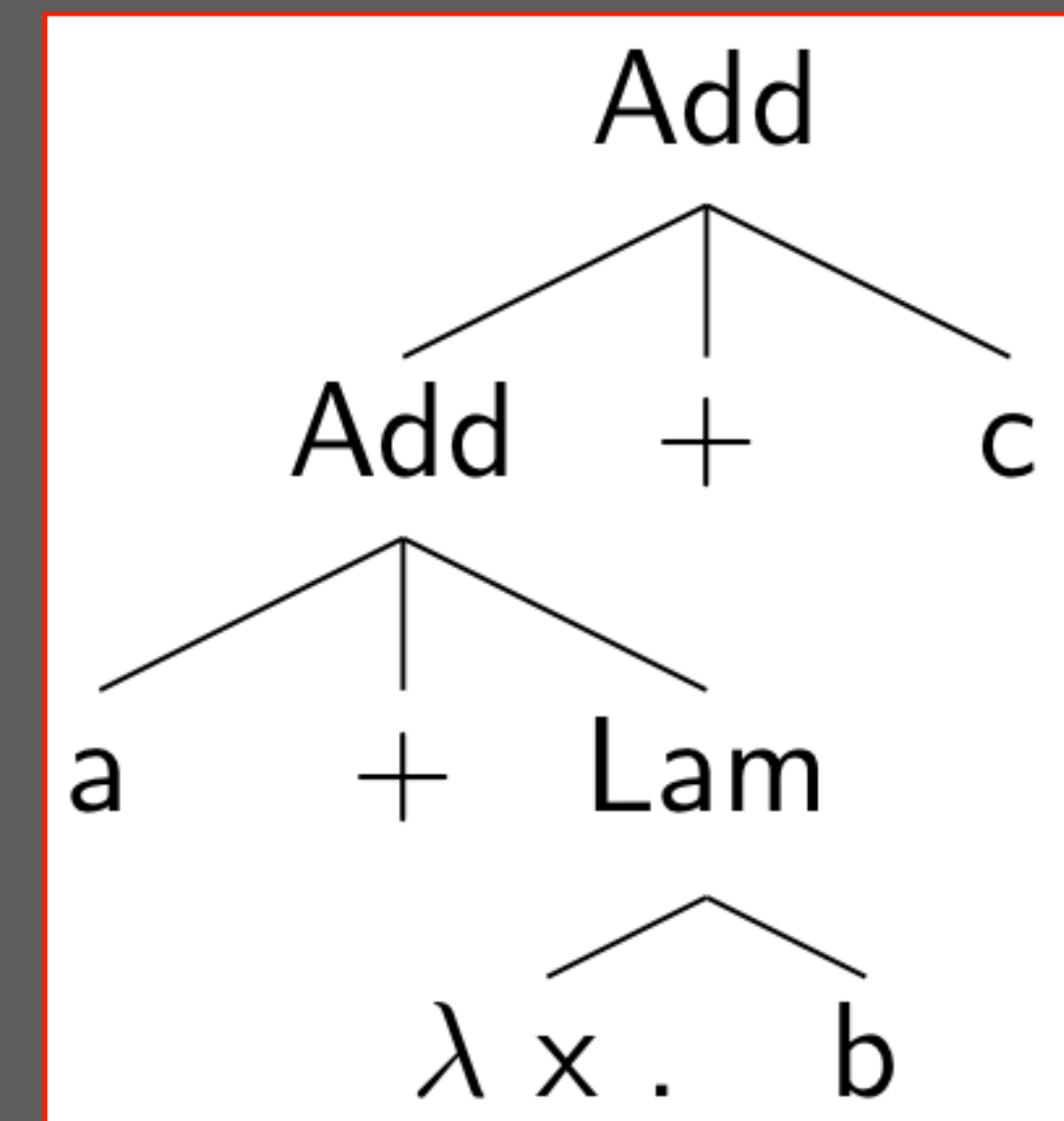
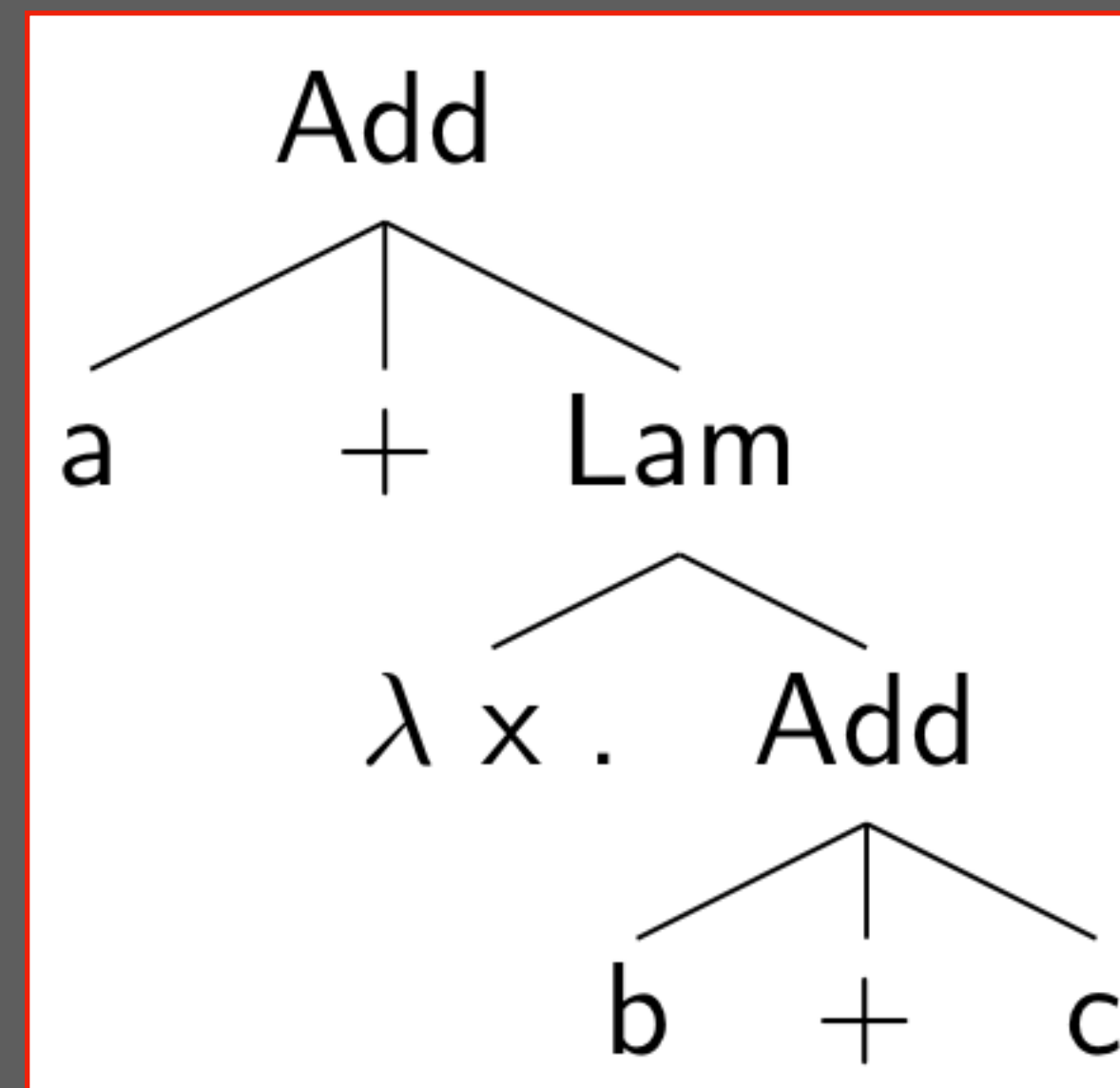
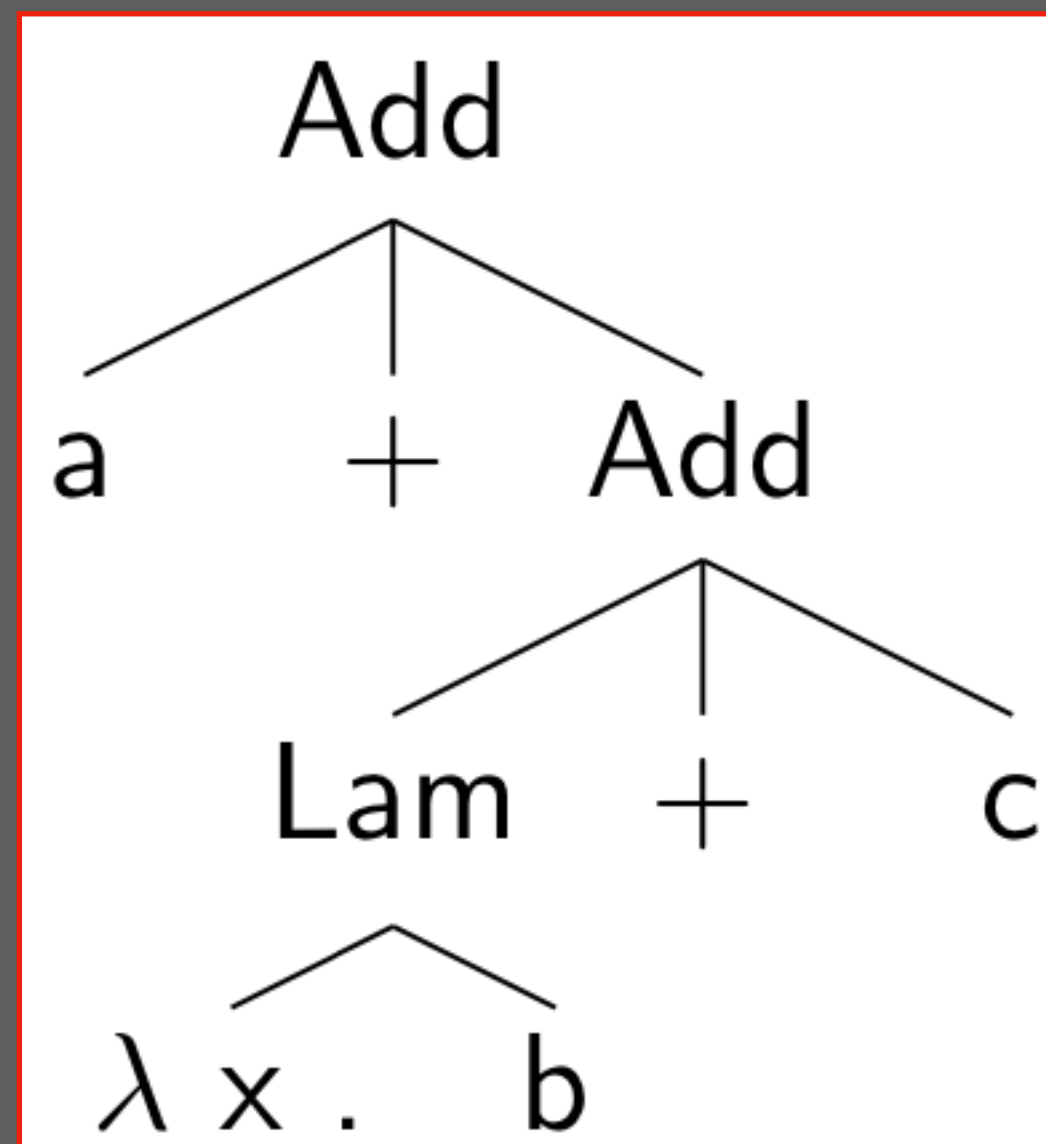


Unsafe for Low Priority Prefix Operators [SDF2]

Conflict
Patterns

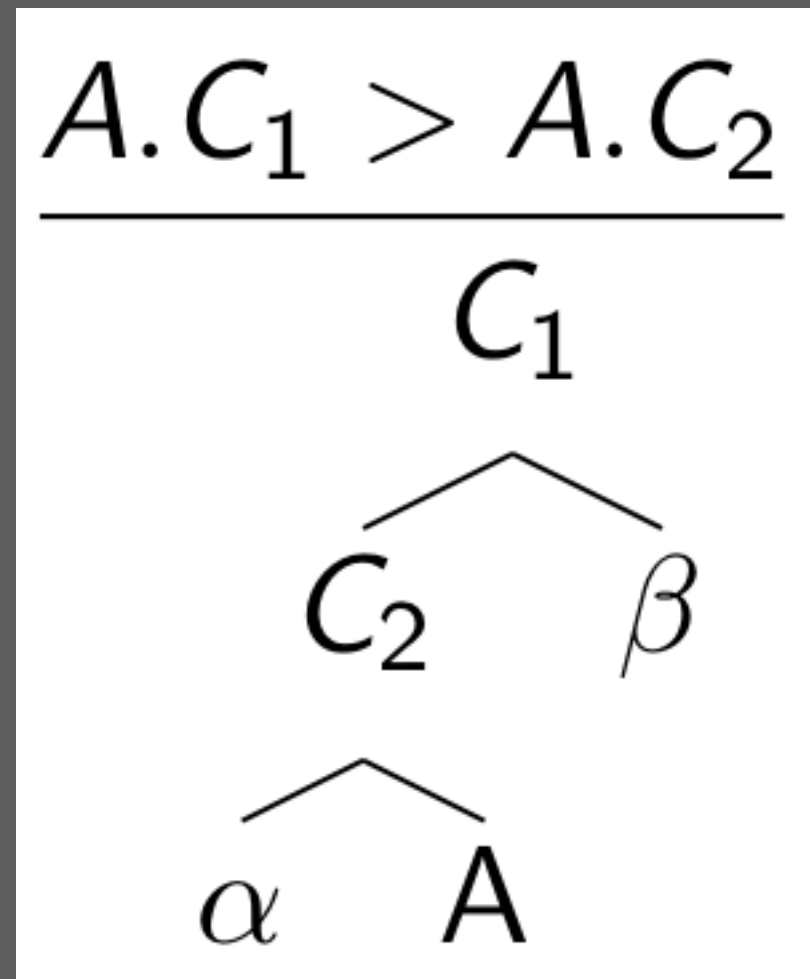


Trees

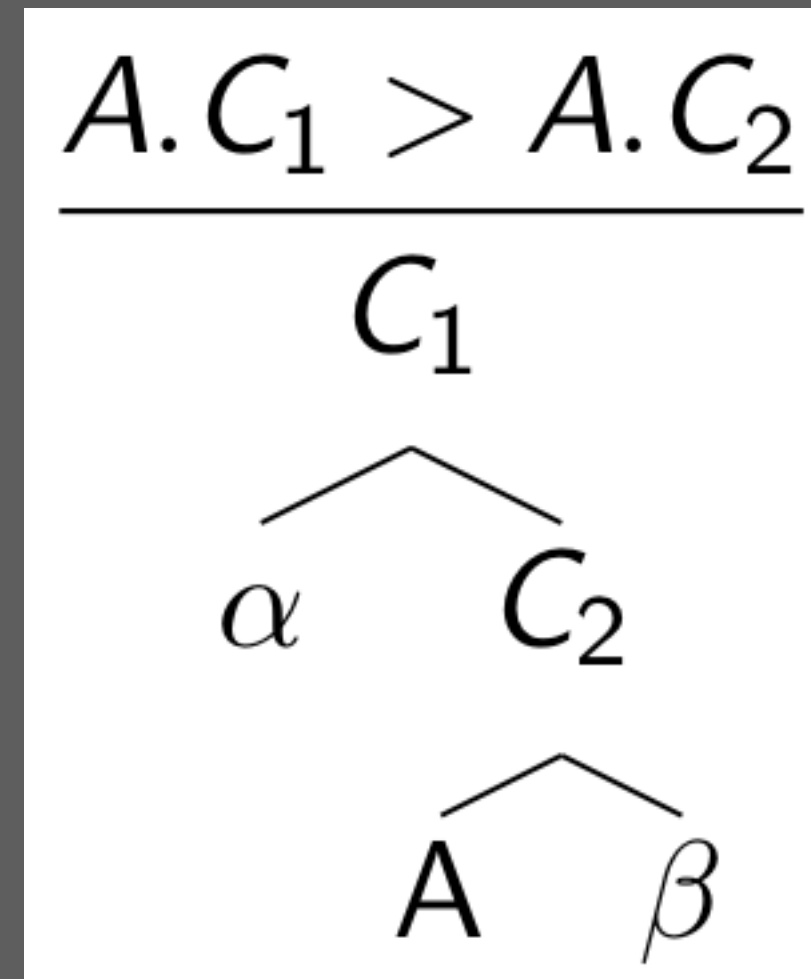


Safe Subtree Exclusion Rules [SDF3 (2019)]

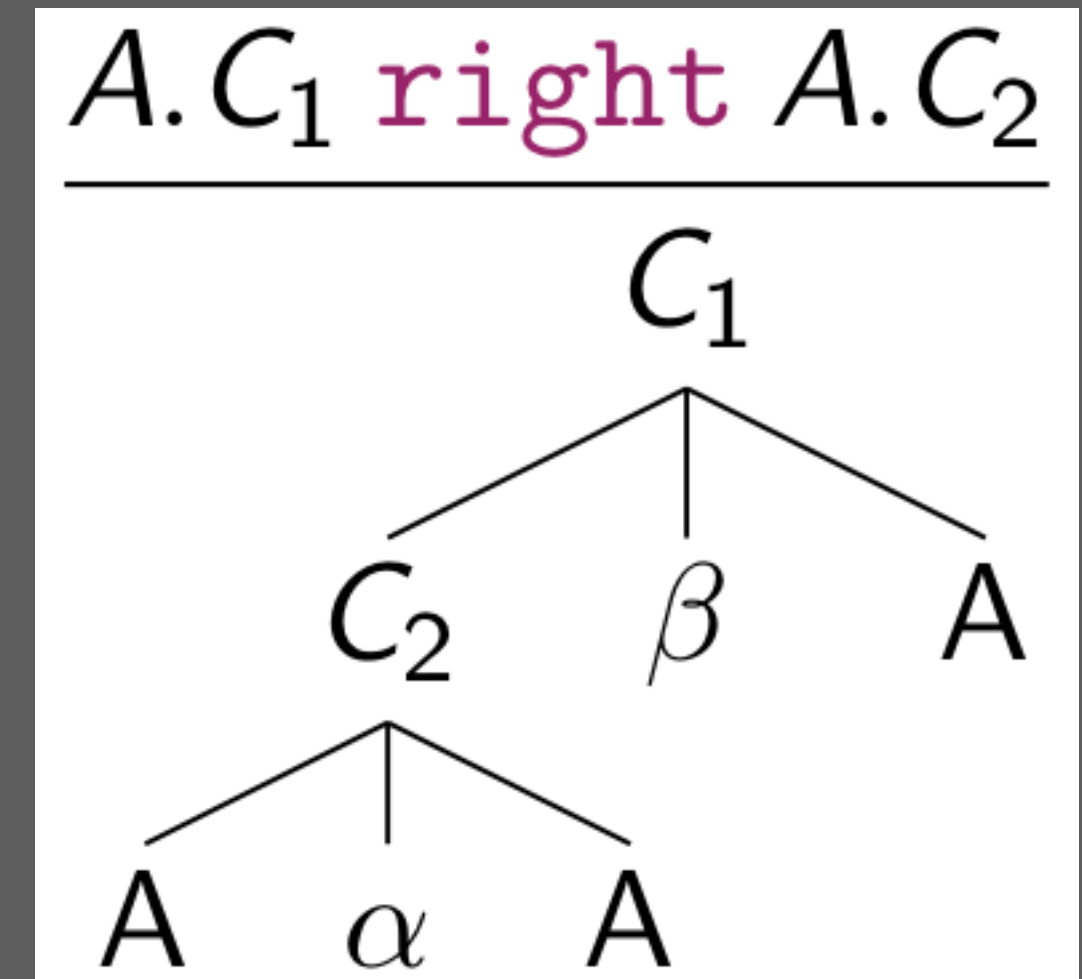
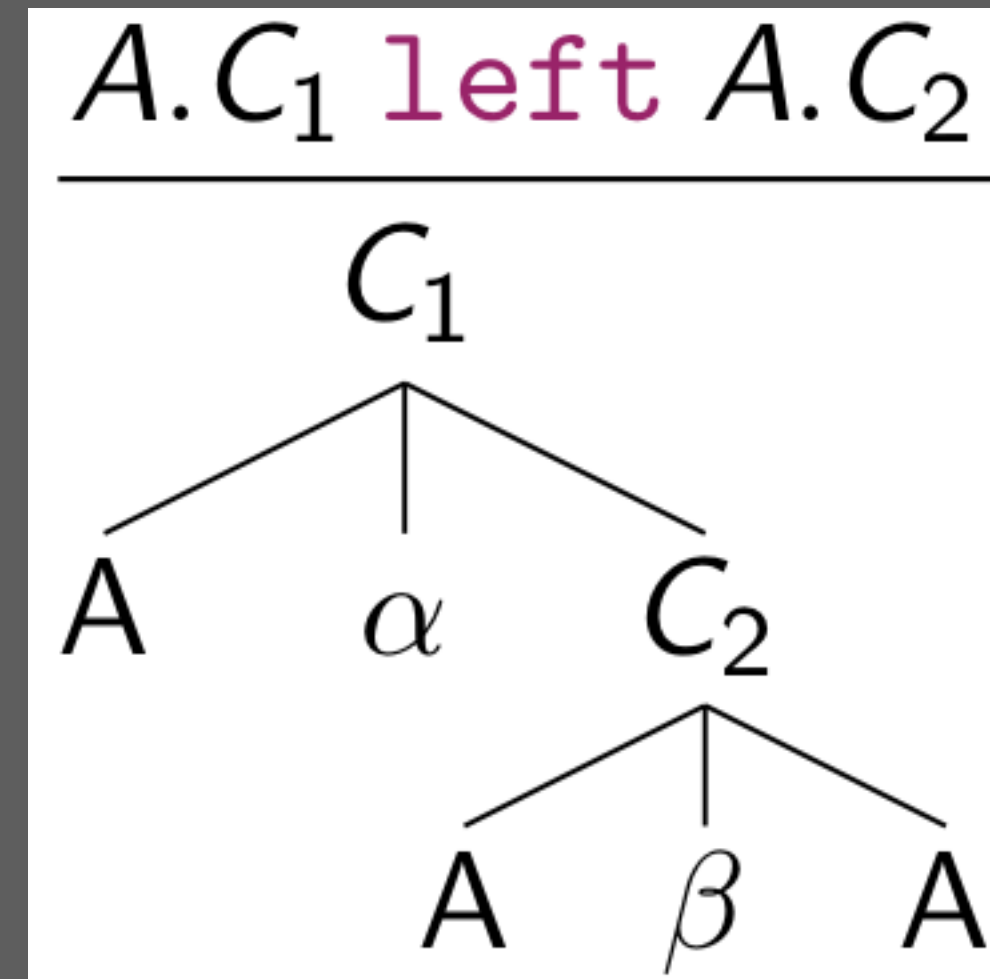
Rules



Right Recursive in
Left Recursive Position

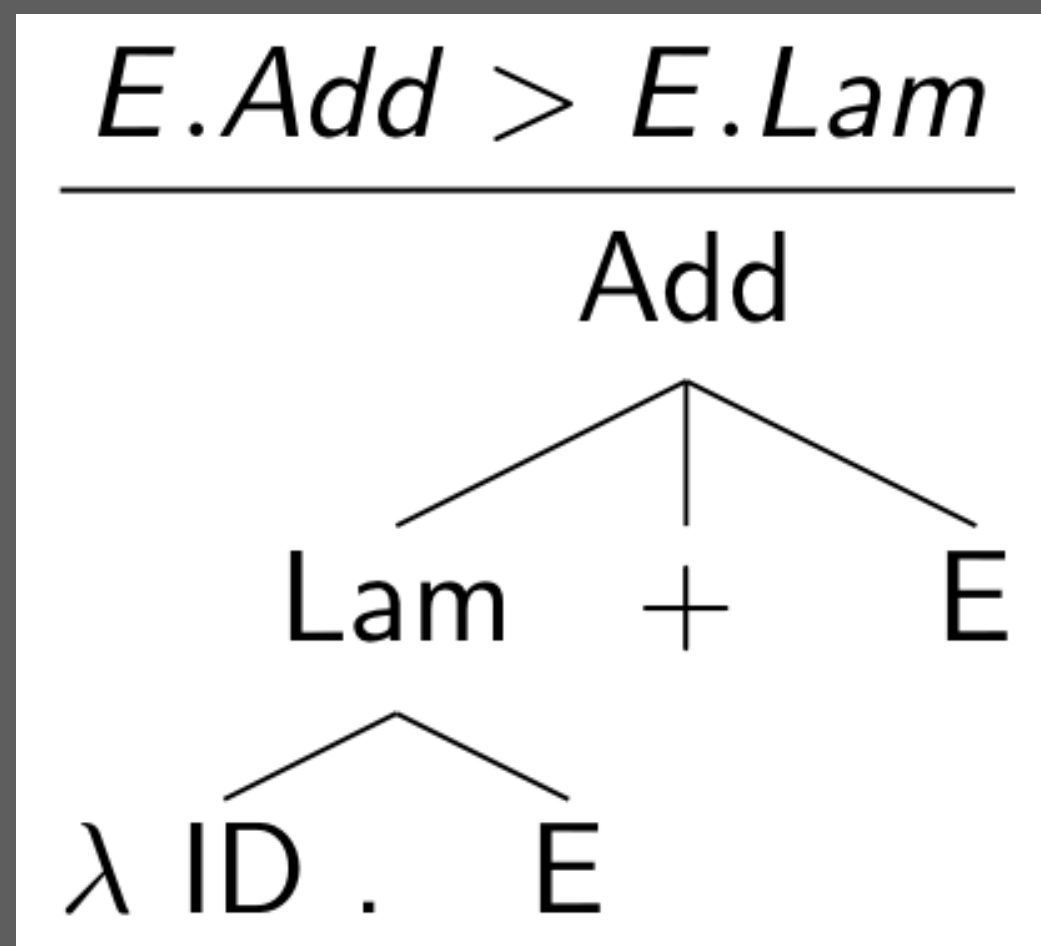


Left Recursive in
Right Recursive Position

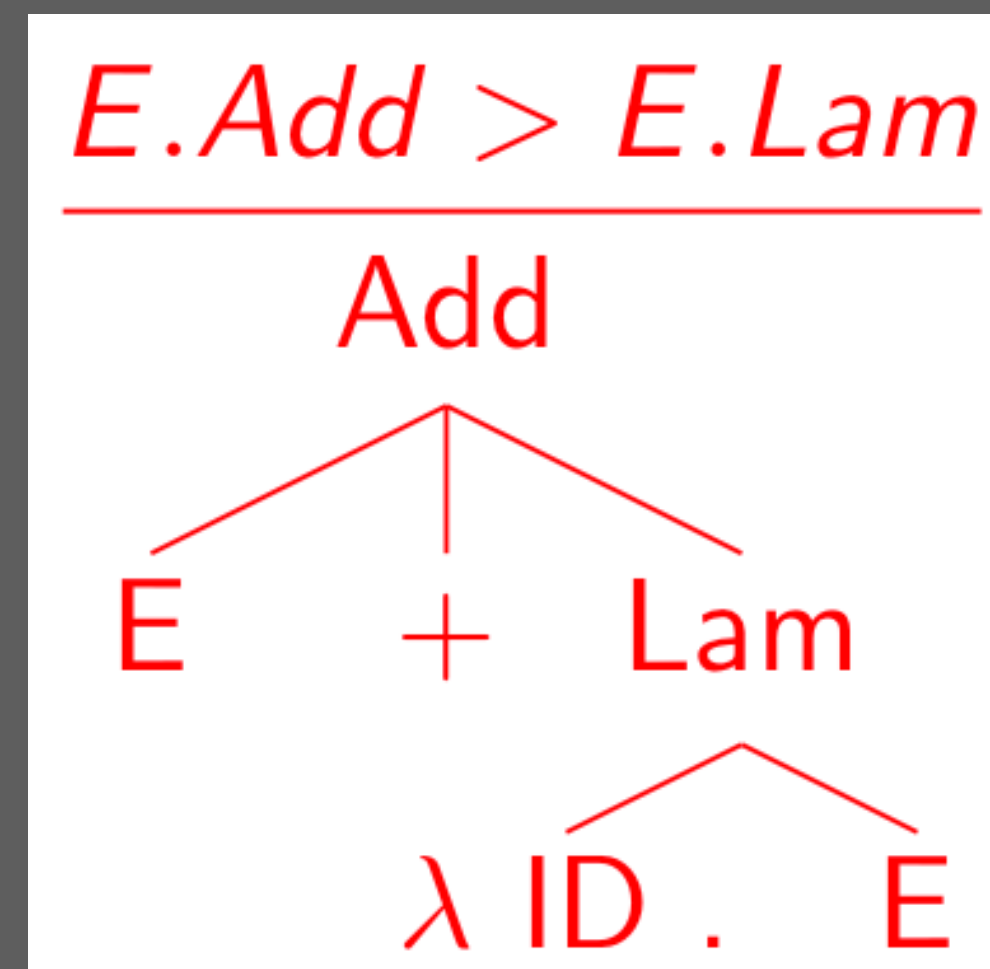


Associativity

Conflict
Patterns



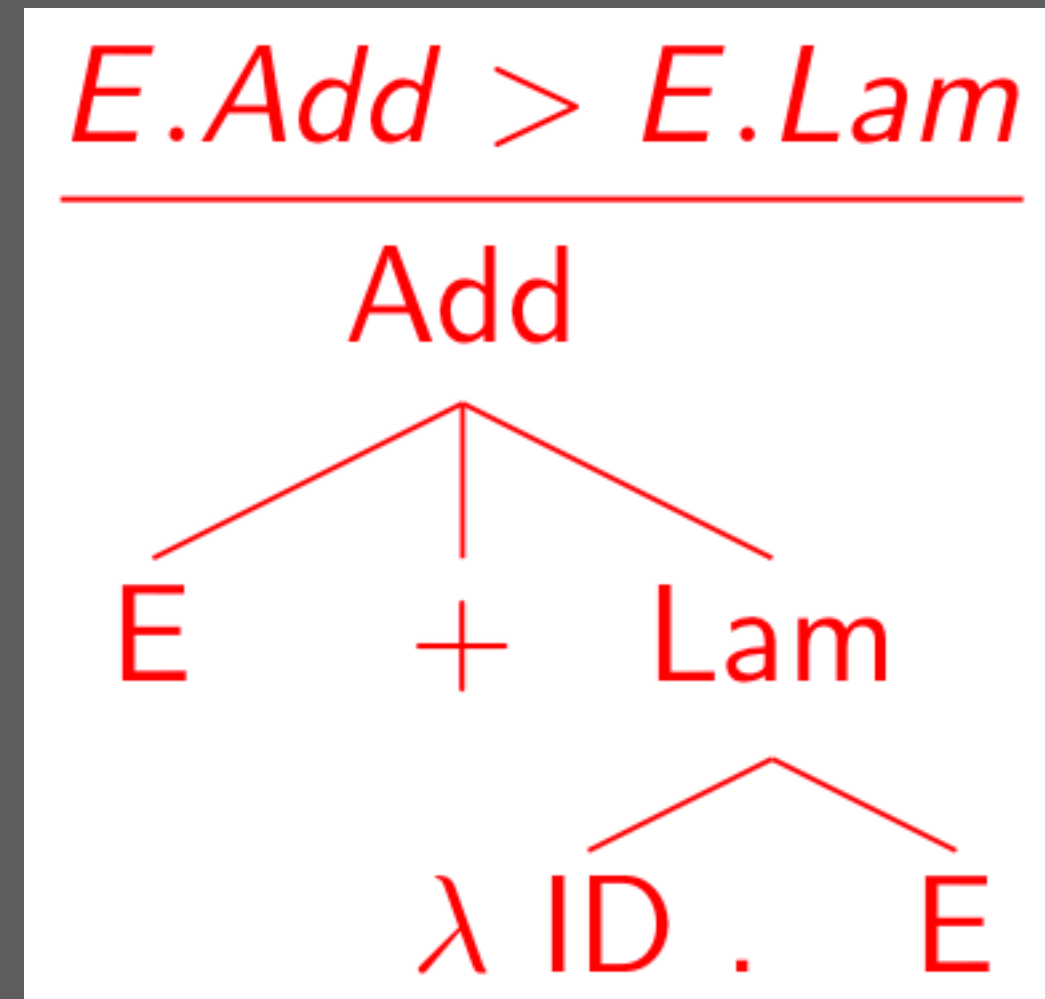
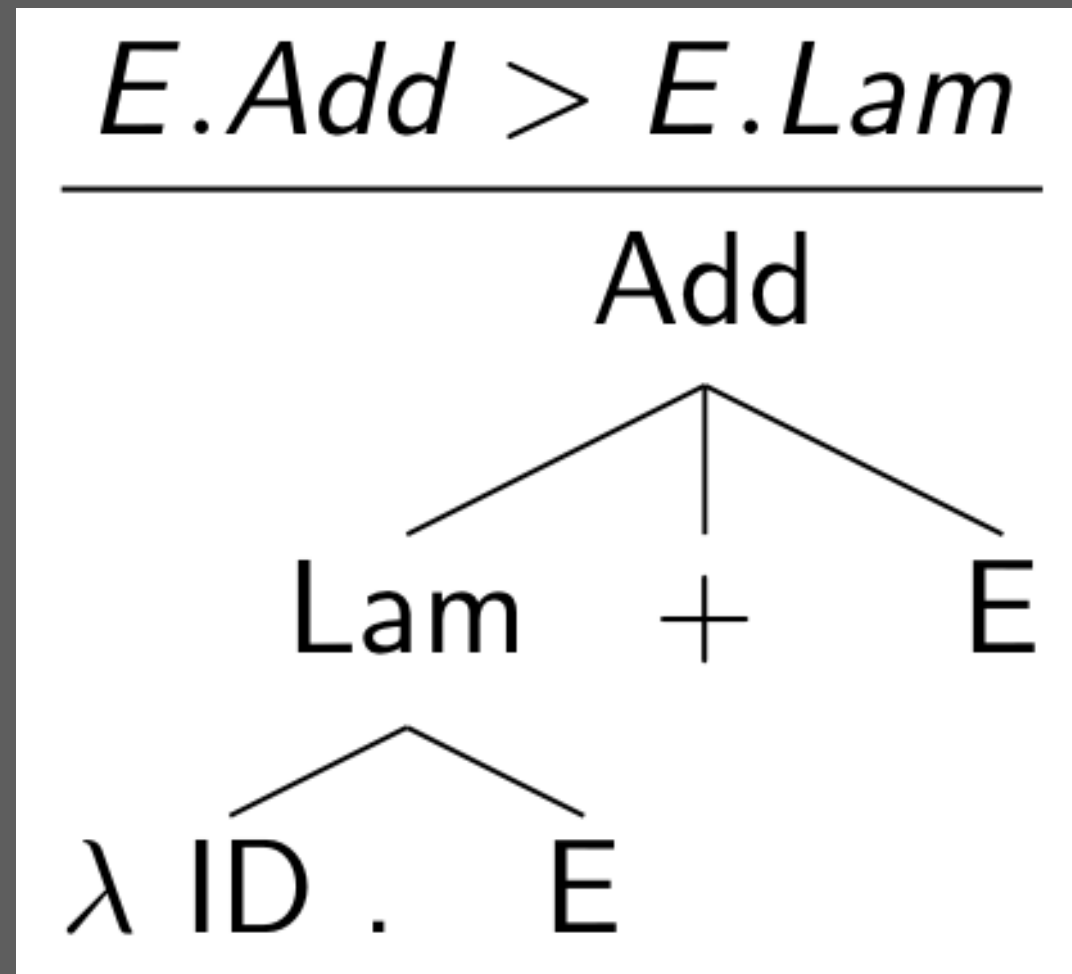
conflict pattern:
\ right recursive



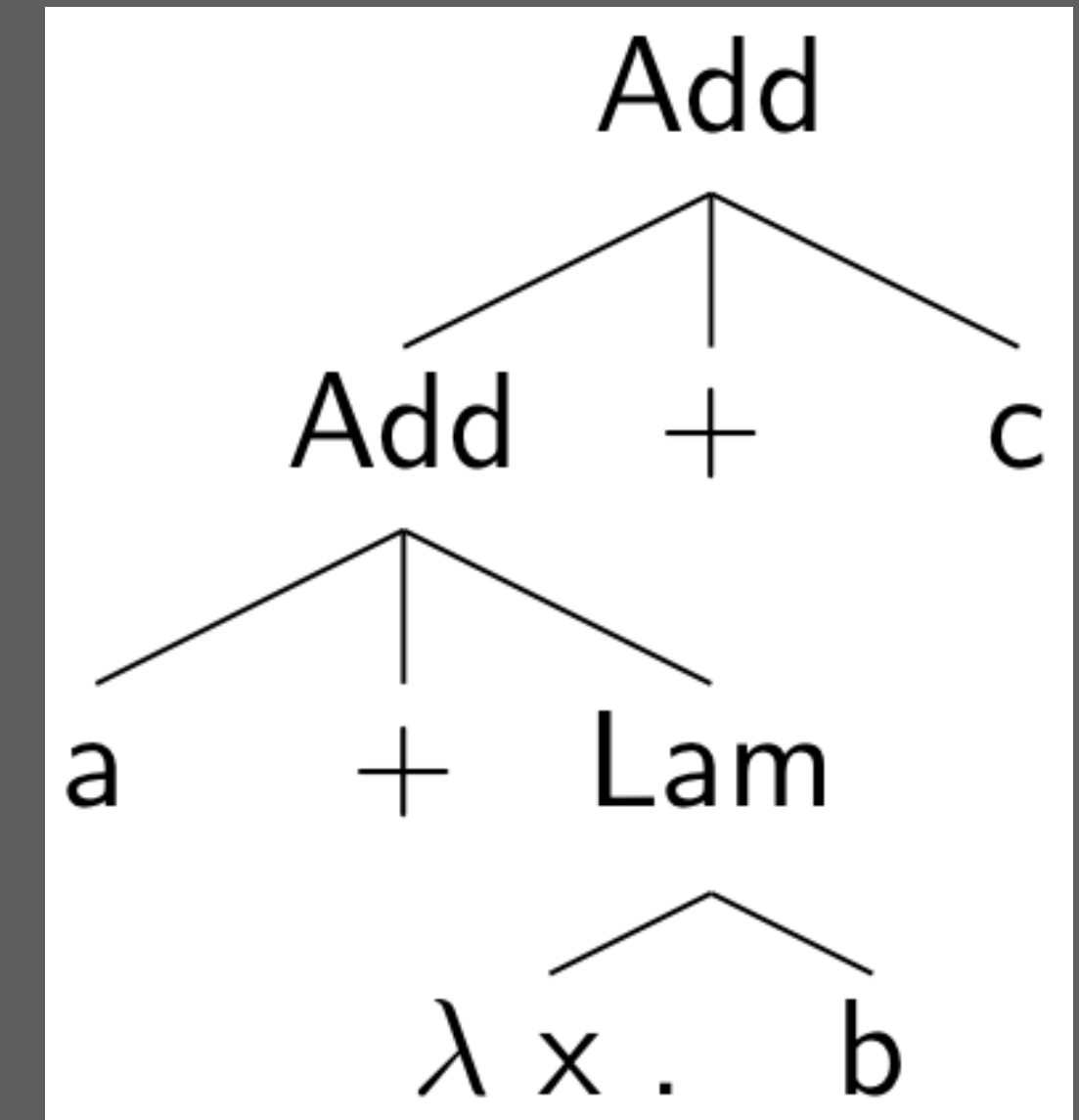
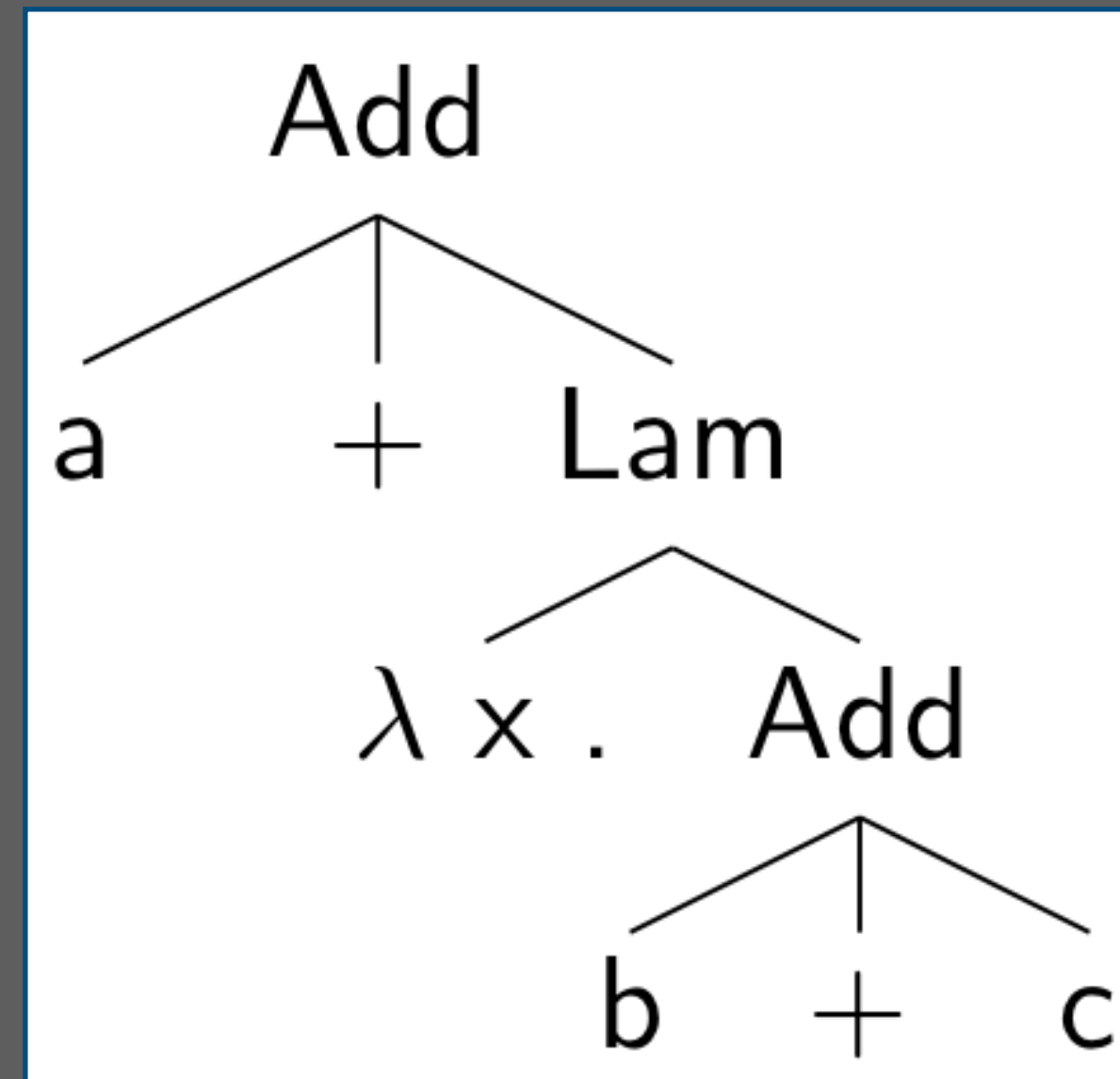
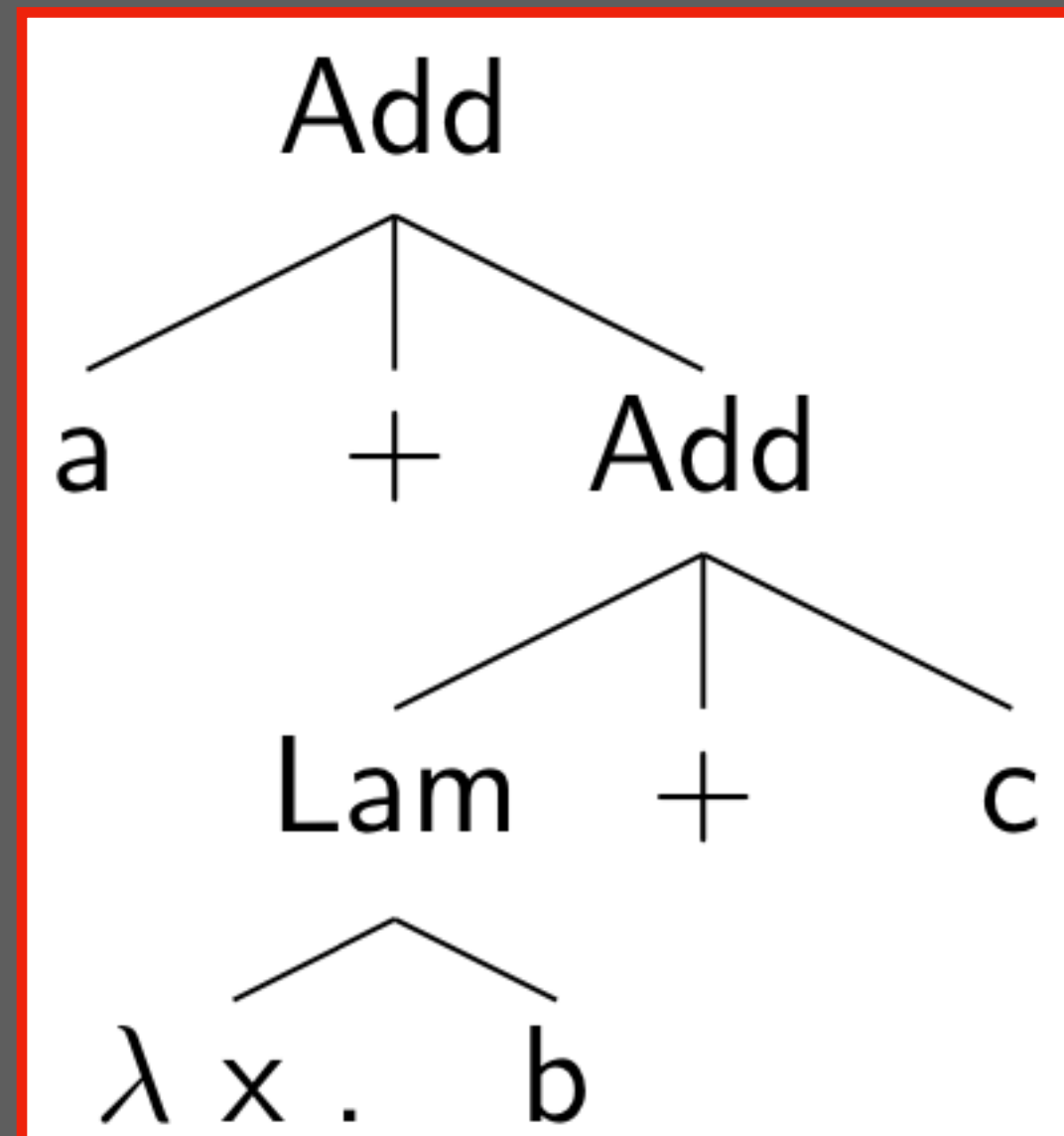
not a conflict pattern:
\ not left recursive

Shallow Interpretation: Safe for Low Priority Prefix Operators

Conflict
Patterns

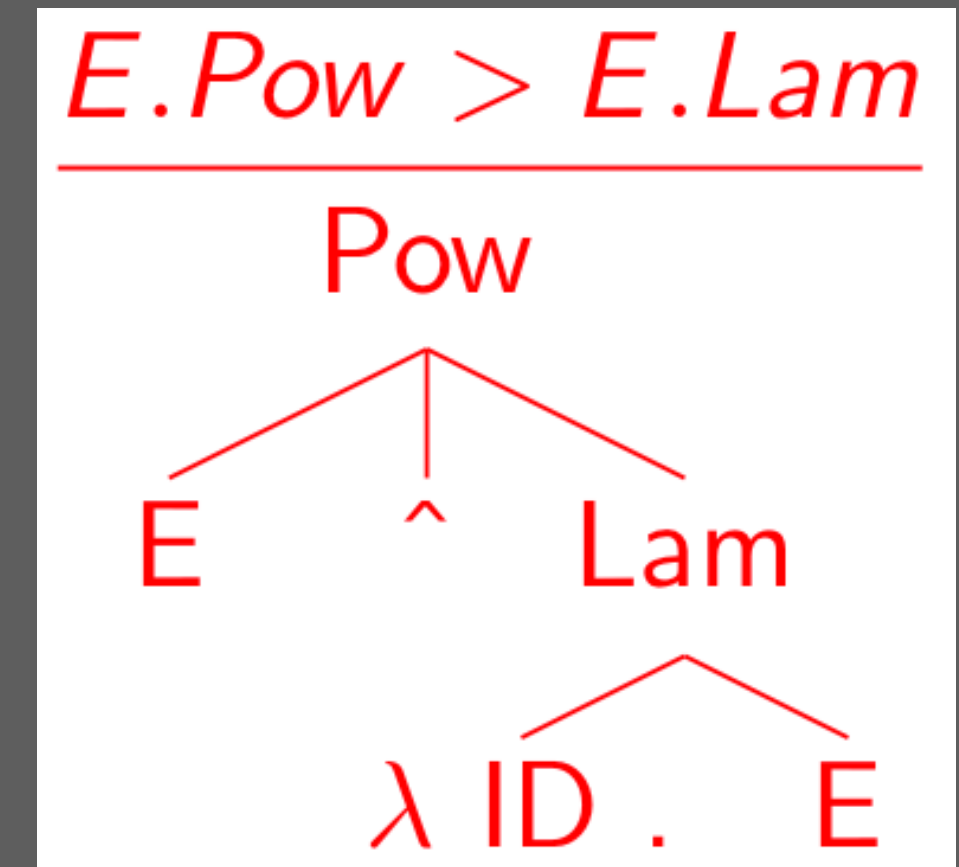
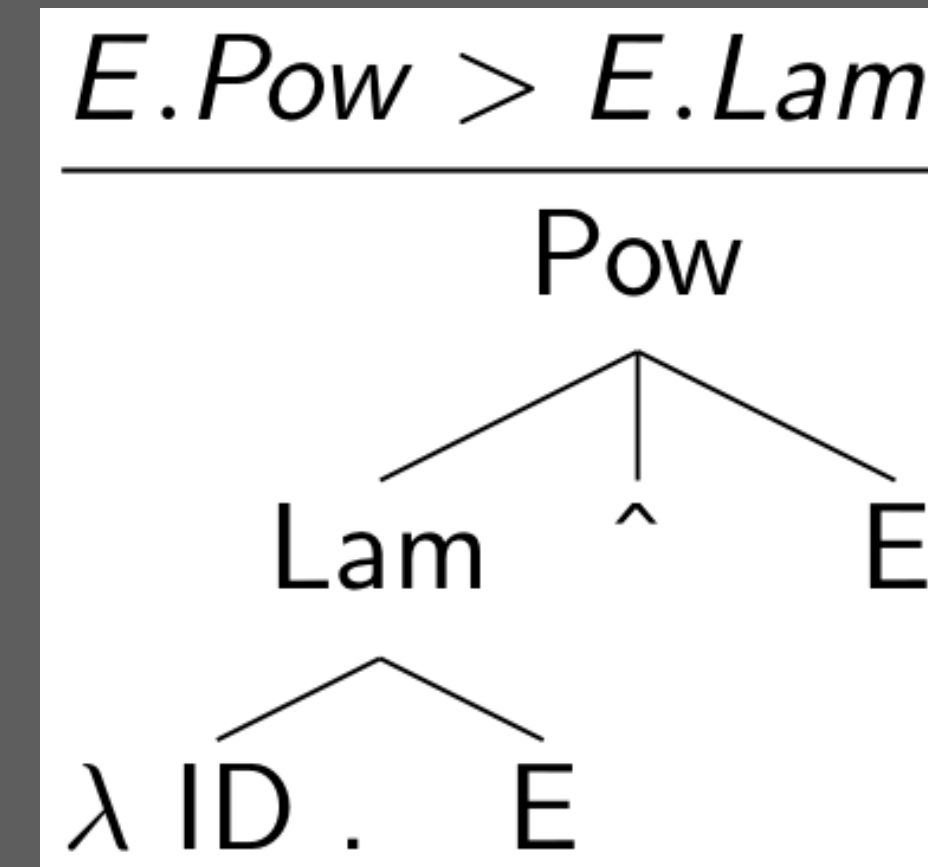
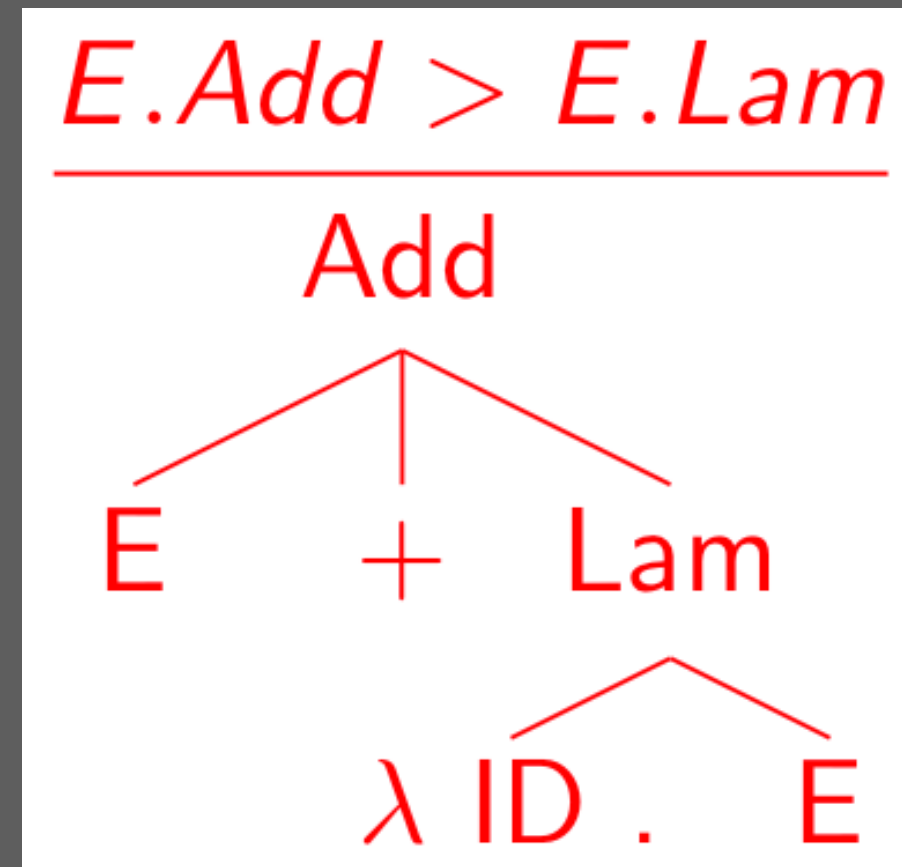
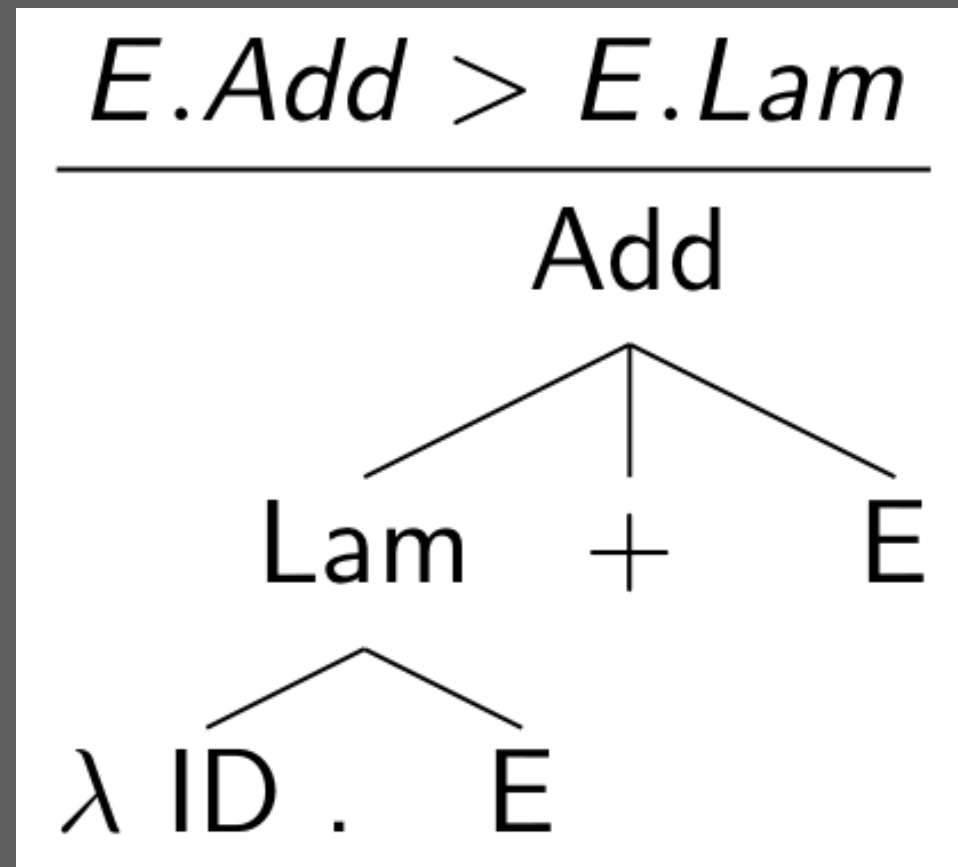


Trees



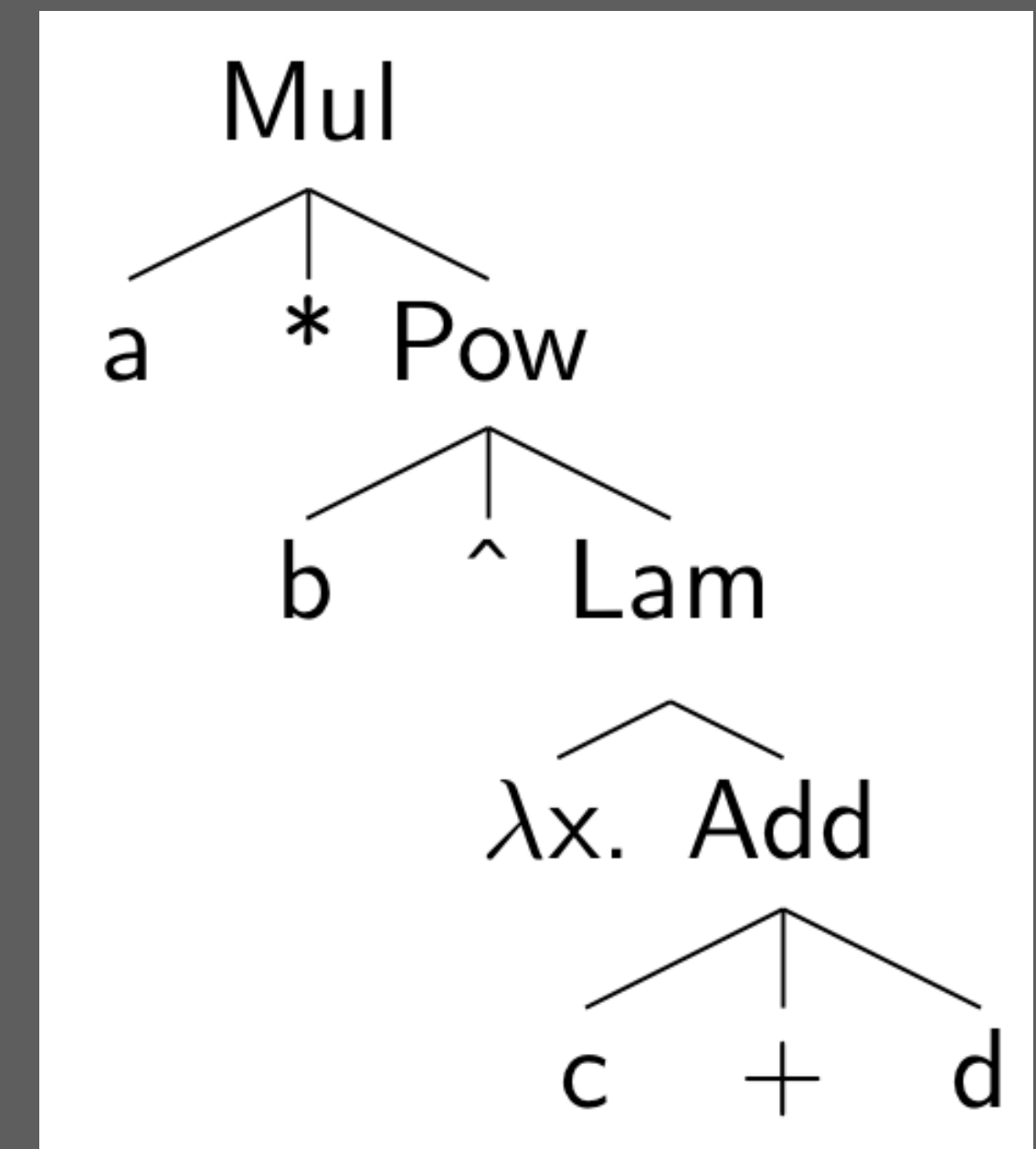
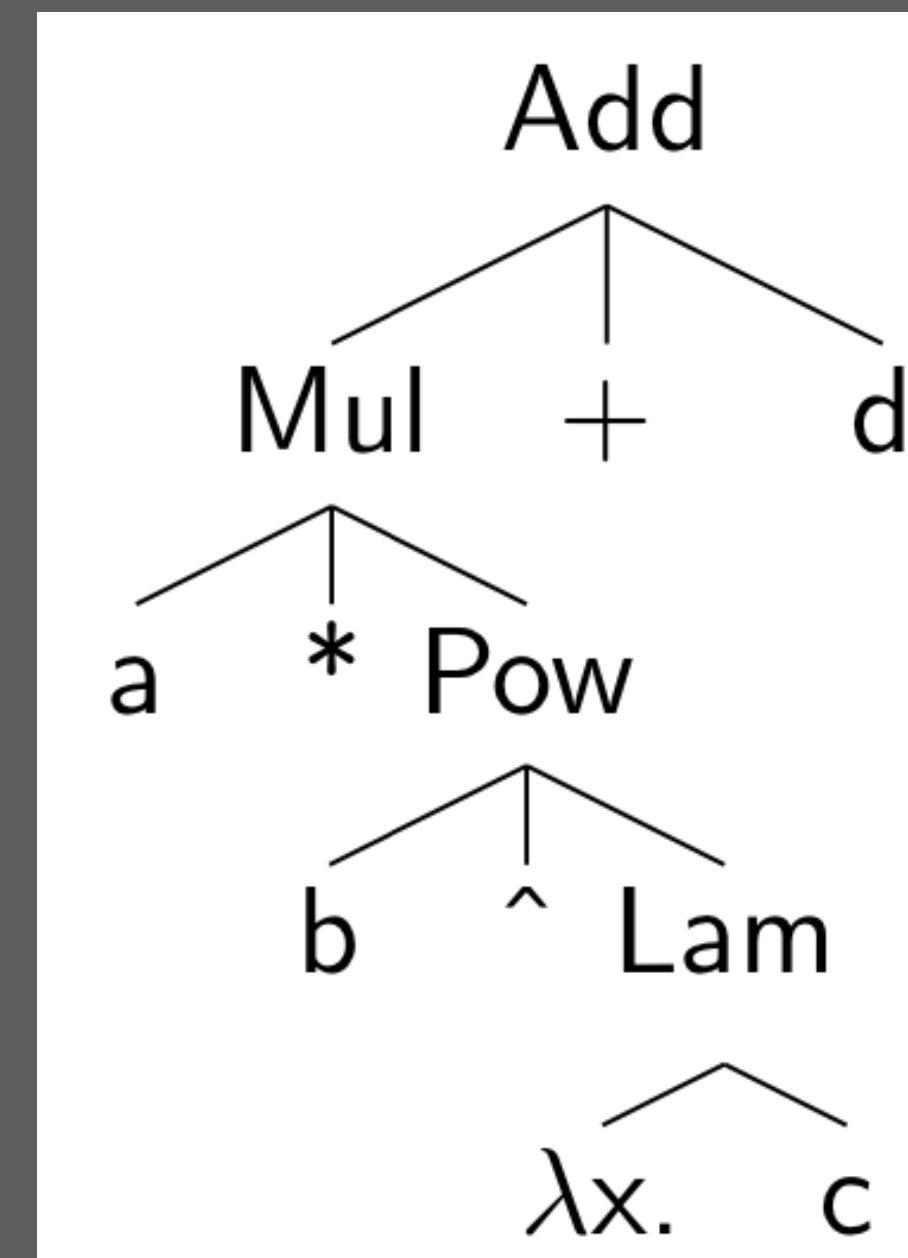
Shallow Interpretation: Incomplete for Low Priority Prefix Operators

Conflict Patterns

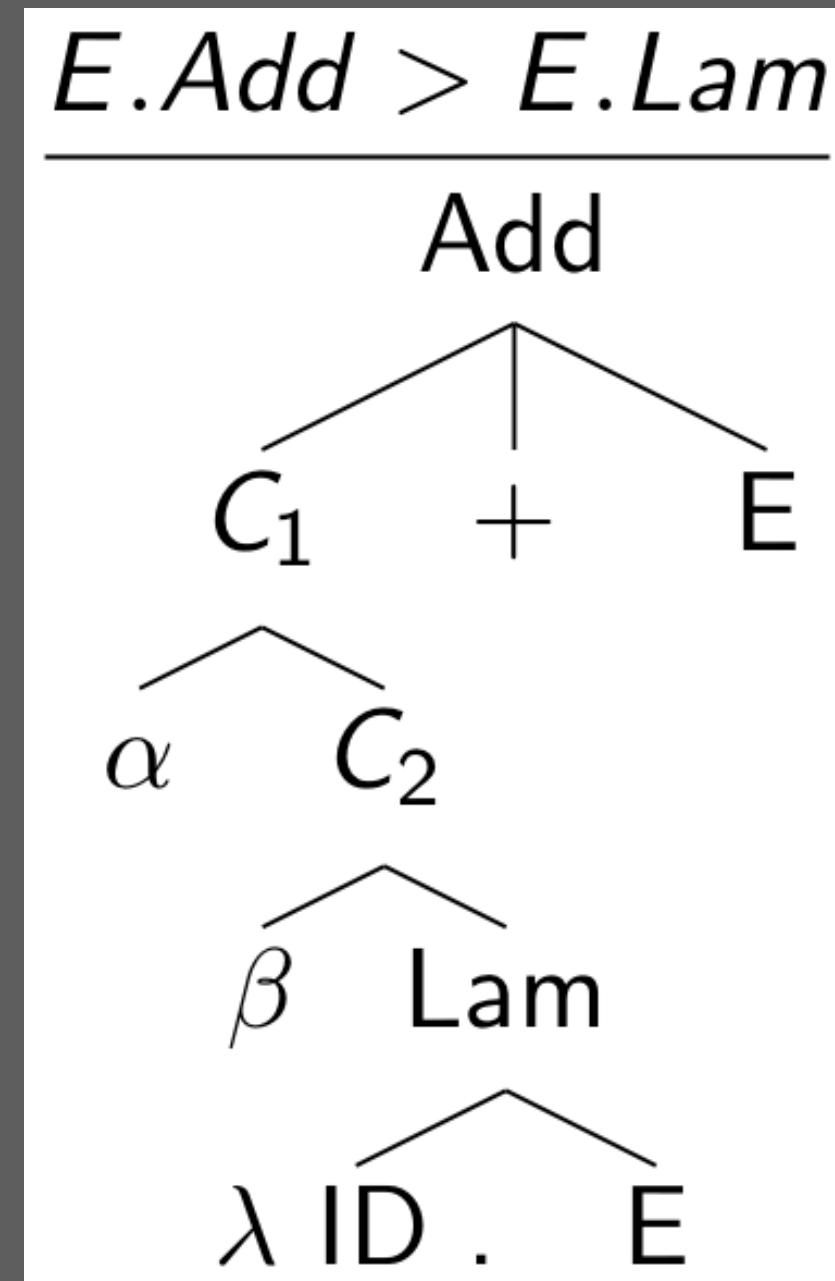
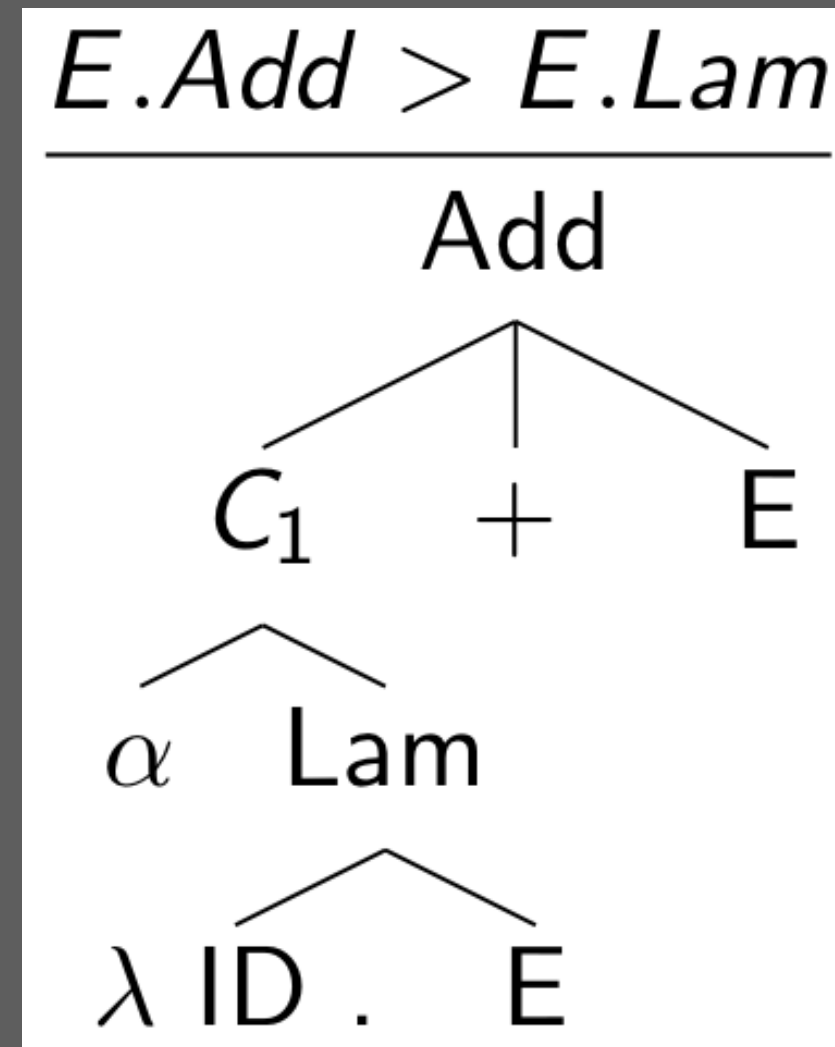
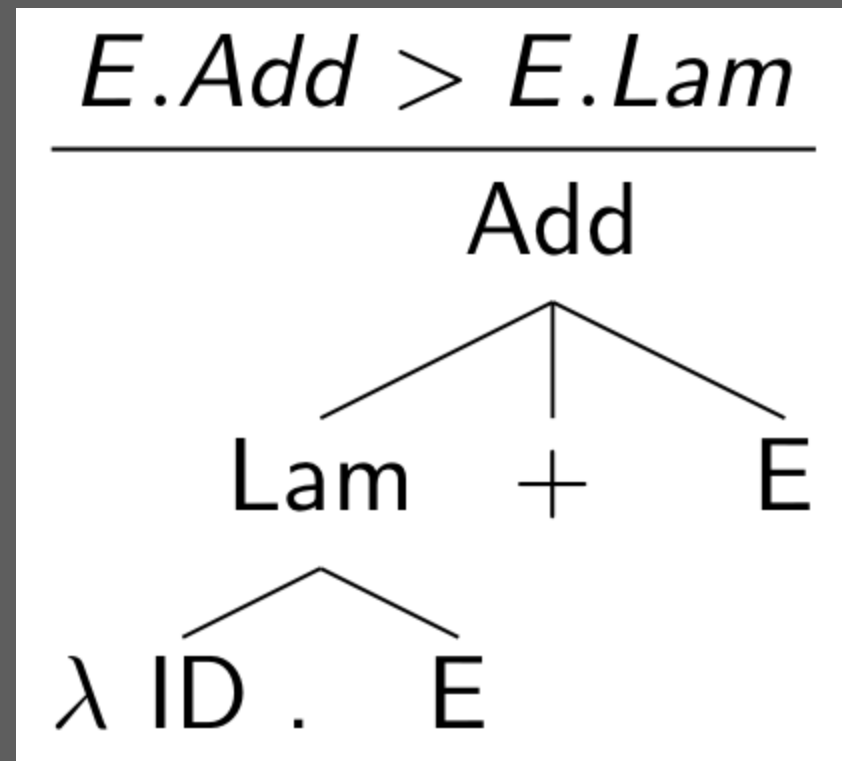


Trees

a * b ^ λ x. c + d

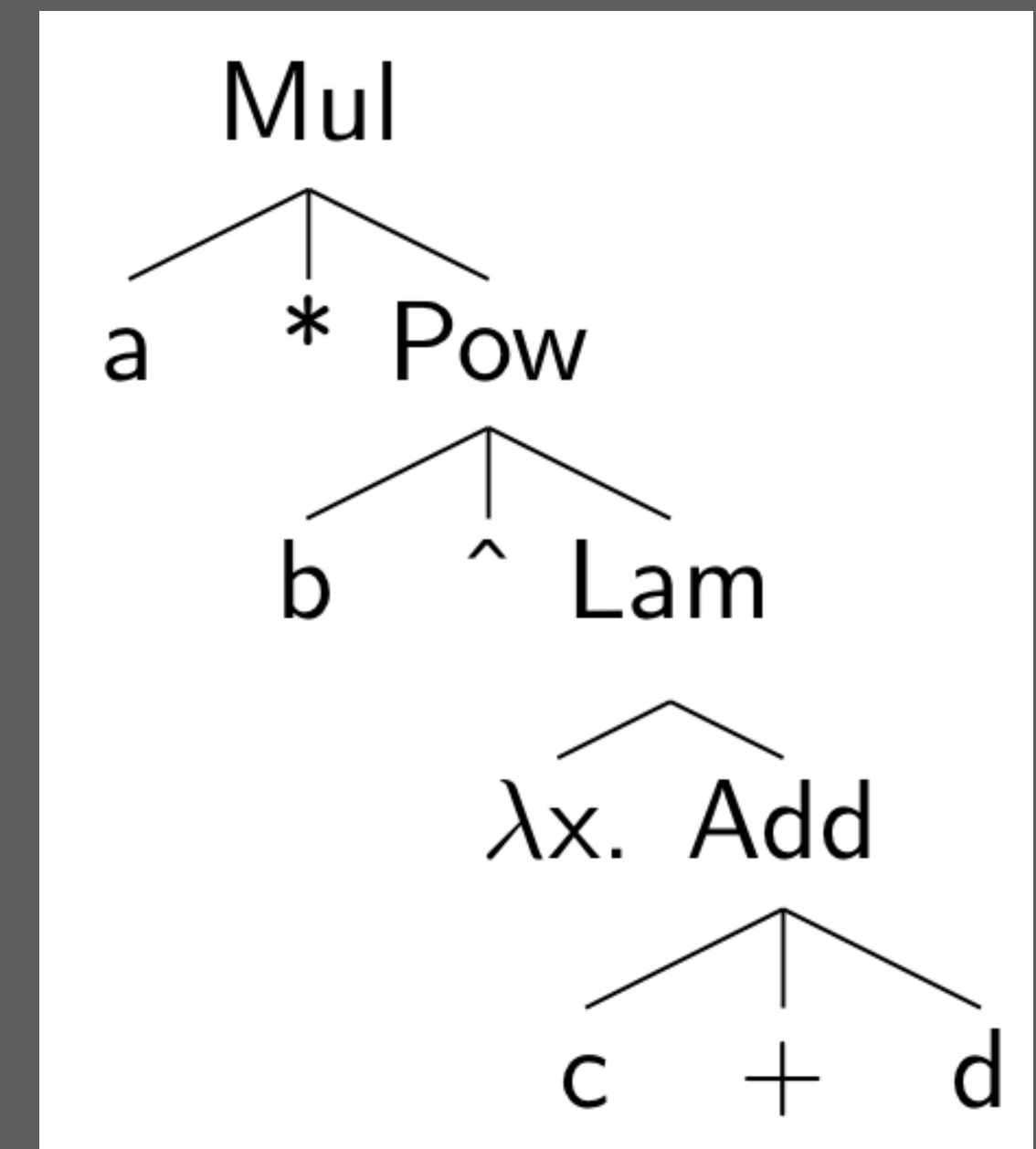
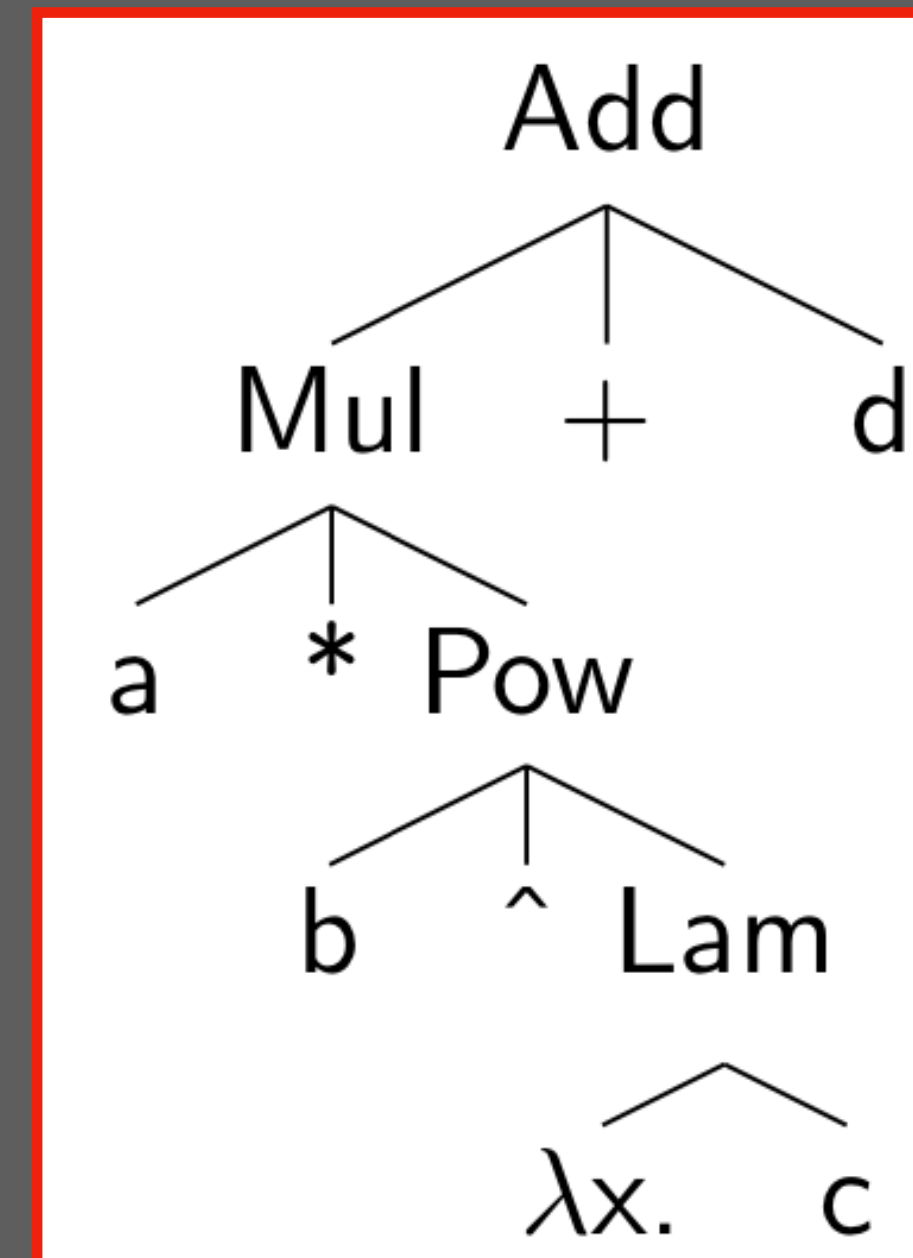


Deep Priority Conflicts: Match Subpattern in Right-Most Subtree

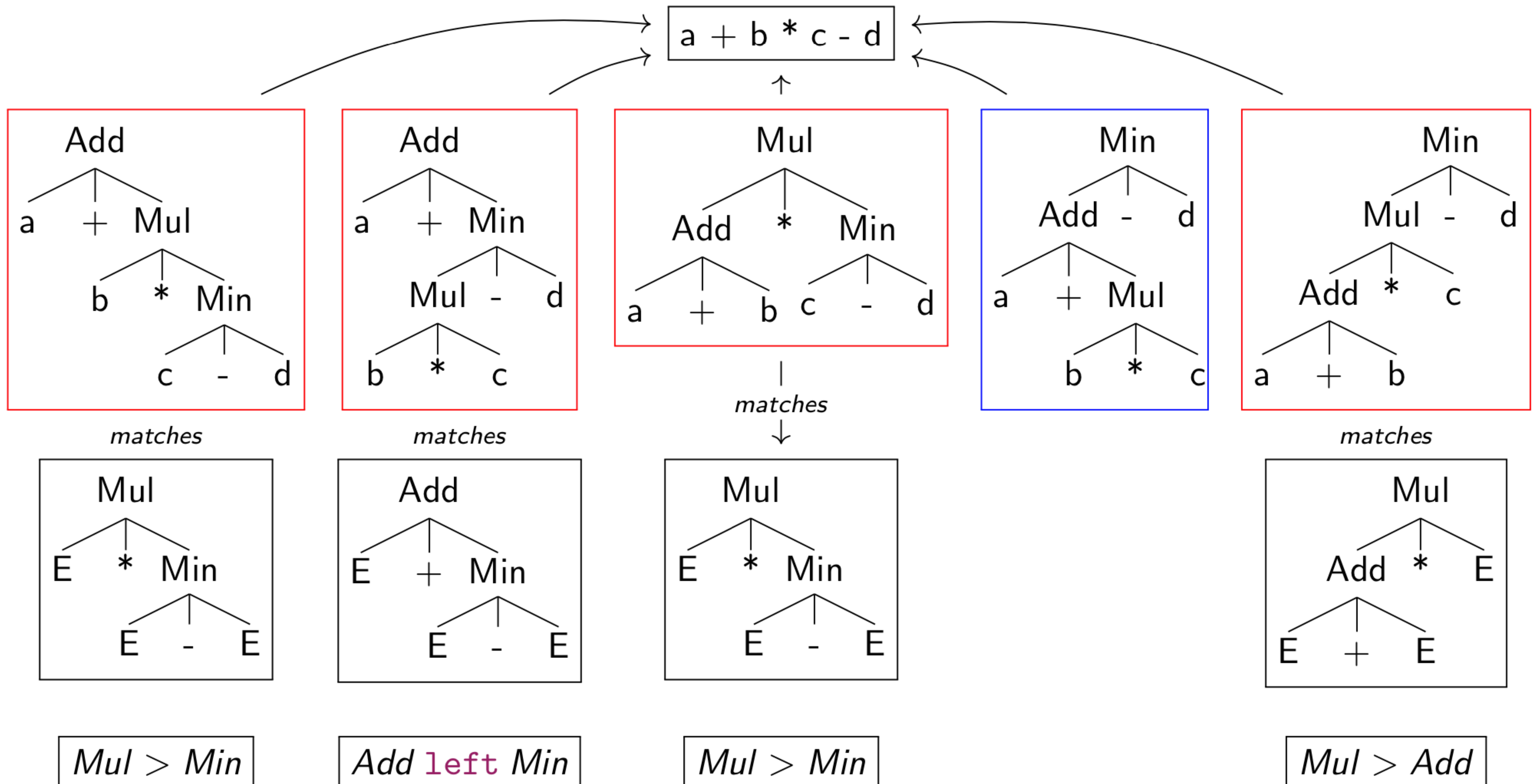


...

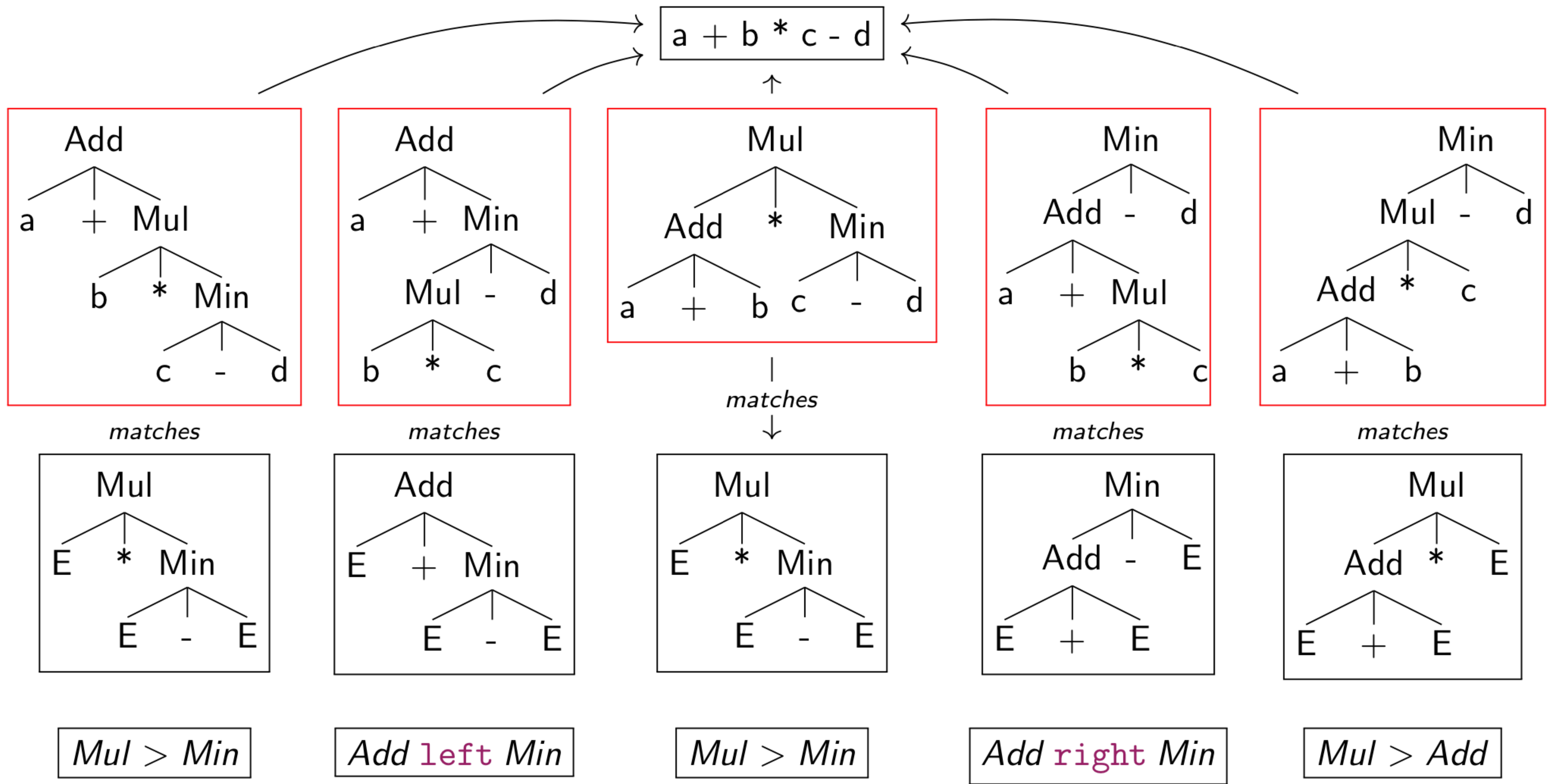
Infinite set of conflict patterns



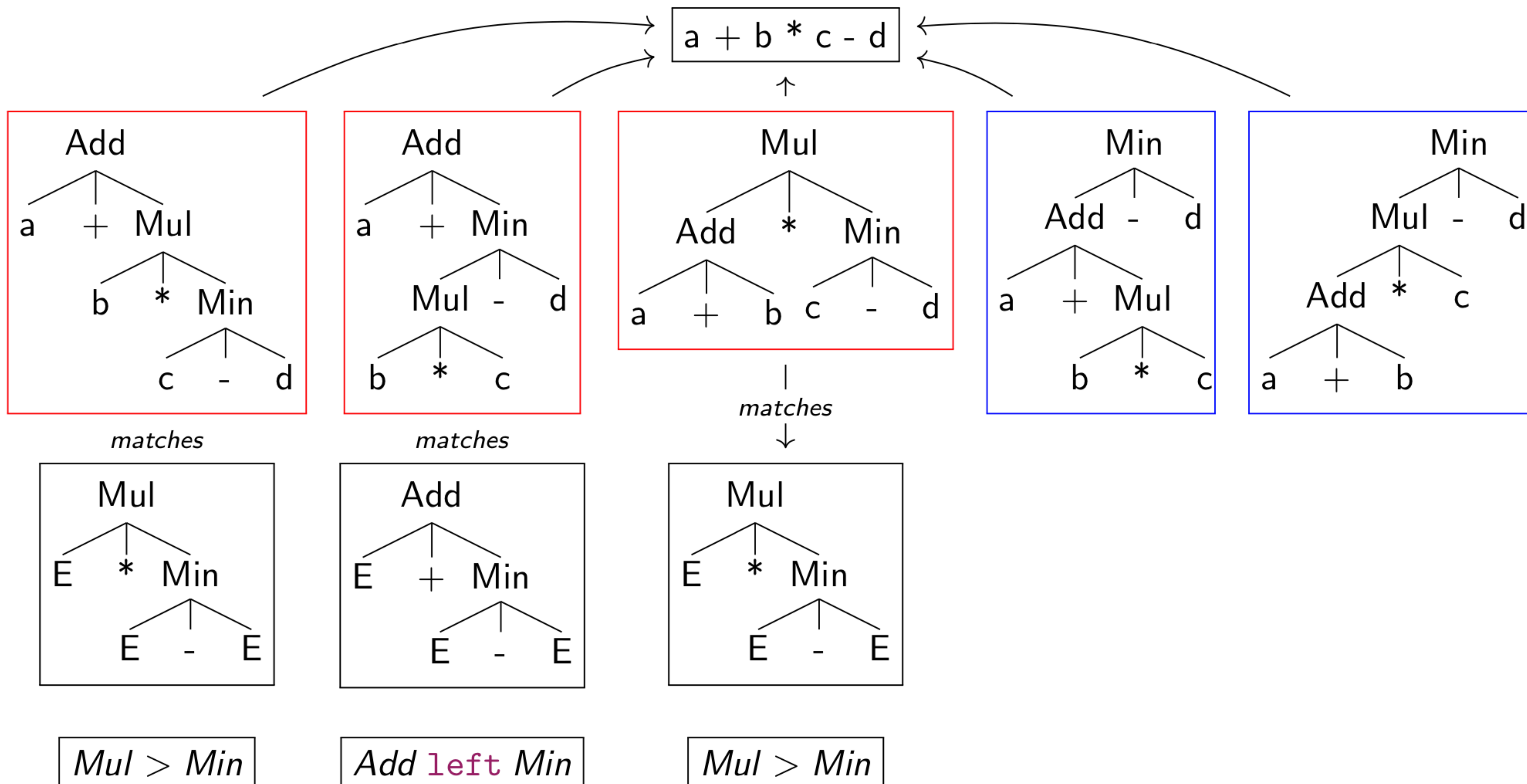
Safe and Complete Disambiguation Rules



Unsafe: Too Many Disambiguation Rules



Incomplete: Too Few Disambiguation Rules



Semantics of Associativity and Priority

What is the semantics of associativity and priority rules?

- subtree exclusion: tree patterns that are forbidden

Is a set of disambiguation rules safe?

- At most one rule for each pair of productions
- + some well-formedness criteria

Is a set of disambiguation rules complete?

- At least one rule for each pair of productions
- + some well-formedness criteria

How to implement?

Grammar Transformations

Grammar Transformations

Why?

- Disambiguation
- For use by a particular parsing algorithm

Transformations

- Eliminating ambiguities
- Eliminating left recursion
- Left factoring

Properties

- Does transformation preserve the language (set of strings, trees)?
- Does transformation preserve the structure of trees?

Ambiguous Expression Grammar

grammar

productions

E.A = E "+" E
E.T = E "*" E
E.M = "-" E
E.B = "(" E ")"
E.V = ID

derivation

$E \Rightarrow * ID \text{ "*" } ID \text{ "+" } ID$

term derivation

E
 $\Rightarrow A(E, E)$
 $\Rightarrow A(T(E, E), E)$
 $\Rightarrow A(T(E, E), E)$
 $\Rightarrow A(T(V(ID), E), E)$
 $\Rightarrow A(T(V(ID), V(ID)), E)$
 $\Rightarrow A(T(V(ID), V(ID)), V(ID))$

term derivation

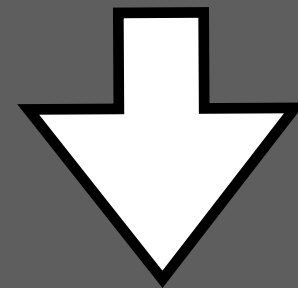
E
 $\Rightarrow T(E, E)$
 $\Rightarrow T(E, E)$
 $\Rightarrow T(V(ID), E)$
 $\Rightarrow T(V(ID), A(E, E))$
 $\Rightarrow T(V(ID), A(V(ID), E))$
 $\Rightarrow T(V(ID), A(V(ID), V(ID)))$

Associativity and Priority Filter Ambiguities

grammar

productions

E.A = E "+" E
E.T = E "*" E
E.M = "-" E
E.B = "(" E ")"
E.V = ID



grammar

productions

E.A = E "+" E {left}
E.T = E "*" E {left}
E.M = "-" E
E.B = "(" E ")"
E.V = ID

priorities

E.M > E.T > E.A

derivation

E \Rightarrow * ID "*" ID "+" ID

term derivation

E
 \Rightarrow A(E, E)
 \Rightarrow A(T(E, E), E)
 \Rightarrow A(T(E, E), E)
 \Rightarrow A(T(V(ID), E), E)
 \Rightarrow A(T(V(ID), V(ID)), E)
 \Rightarrow A(T(V(ID), V(ID)), V(ID))

~~term derivation~~

~~E
 \Rightarrow T(E, E)
 \Rightarrow T(E, E)
 \Rightarrow T(V(ID), E)
 \Rightarrow T(V(ID), A(E, E))
 \Rightarrow T(V(ID), A(V(ID), E))
 \Rightarrow T(V(ID), A(V(ID), V(ID)))~~

Define Associativity and Priority by Transformation

grammar

productions

$E.A = E "+" E$ {left}

$E.T = E "*" E$ {left}

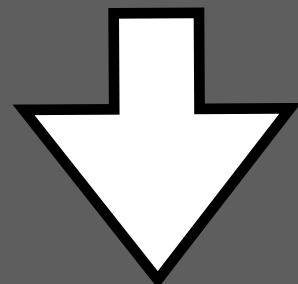
$E.M = "-" E$

$E.B = "(" E ")"$

$E.V = ID$

priorities

$E.M > E.T > E.A$



grammar

productions

$E.A = E "+" T$

$E = T$

$T.T = T "*" F$

$T = F$

$F.V = ID$

$F.B = "(" E ")"$

derivation

$E \Rightarrow * ID "*" ID "+" ID$

term derivation

E

$\Rightarrow A(E, E)$

$\Rightarrow A(T(E, E), E)$

$\Rightarrow A(T(E, E), E)$

$\Rightarrow A(T(V(ID), E), E)$

$\Rightarrow A(T(V(ID), V(ID)), E)$

$\Rightarrow A(T(V(ID), V(ID)), V(ID))$

term derivation

E

$\Rightarrow T(E, E)$

$\Rightarrow T(E, E)$

$\Rightarrow T(V(ID), E)$

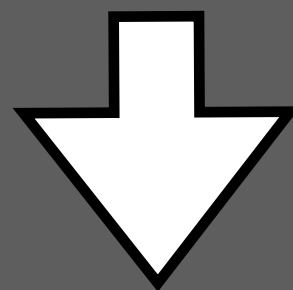
$\Rightarrow T(V(ID), A(E, E))$

$\Rightarrow T(V(ID), A(V(ID), E))$

$\Rightarrow T(V(ID), A(V(ID), V(ID)))$

Define Associativity and Priority by Transformation

```
grammar
  productions
    E.A = E "+" E {left}
    E.T = E "*" E {left}
    E.M = "-" E
    E.B = "(" E ")"
    E.V = ID
  priorities
    E.M > E.T > E.A
```



```
grammar
  productions
    E.A = E "+" T
    E    = T
    T.T = T "*" F
    T    = F
    F.V = ID
    F.M = "-" F
    F.B = "(" E ")"
```

Define new non-terminal for each priority level:
E, T, F

Add 'injection' productions to include priority level n+1 in n:
E = T
T = F

Change head of production to reflect priority level
T = T "*" F

Transform productions
Left: E = E "+" T
Right: E = T "+" E

Dangling Else Grammar

grammar

sorts S E

productions

S.If = if E then S

S.IfE = if E then S else S

S = other

derivation

$S \Rightarrow^*$ if E1 then S1 else if E2 then S2 else S3

term derivation

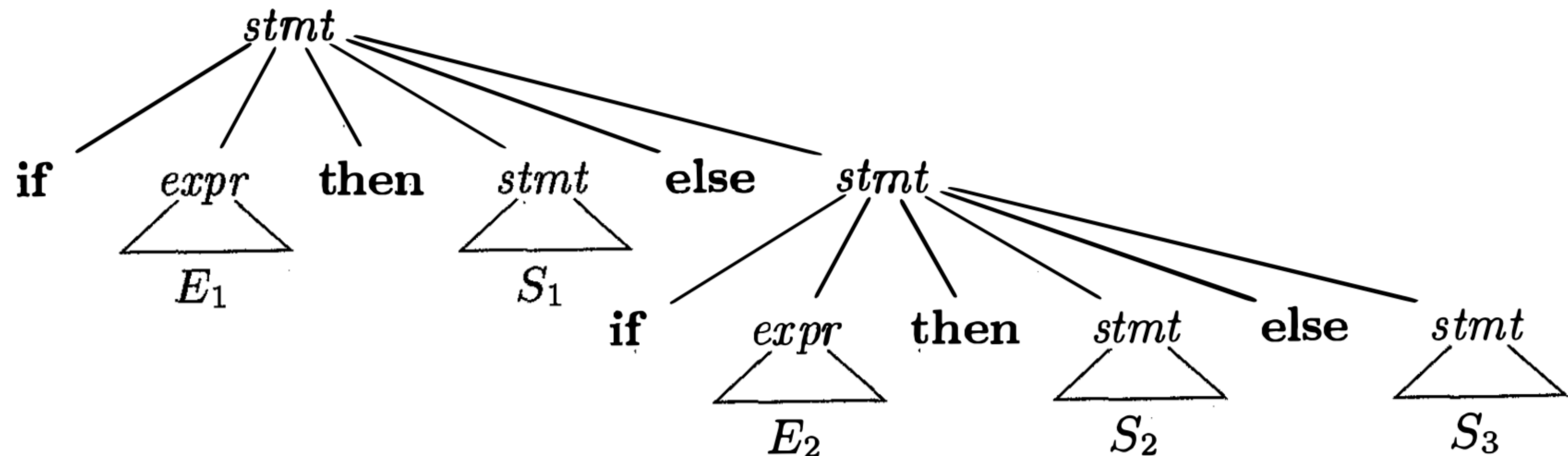
$S \Rightarrow^*$ IfE(E1, S1, IfE(E2, S2, S3))

term derivation

S

\Rightarrow IfE(E1, S1, S)

\Rightarrow IfE(E1, S1, IfE(E2, S2, S3))



Dangling Else Grammar is Ambiguous

grammar

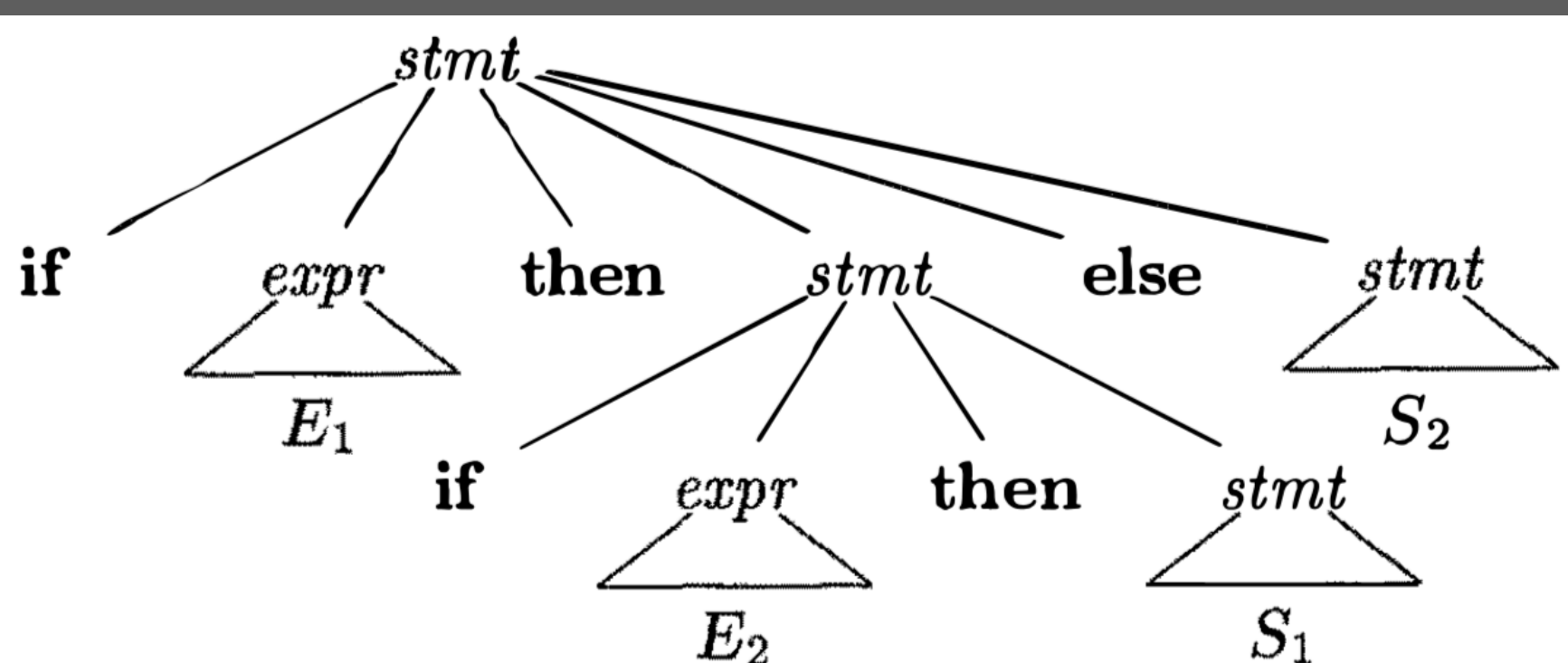
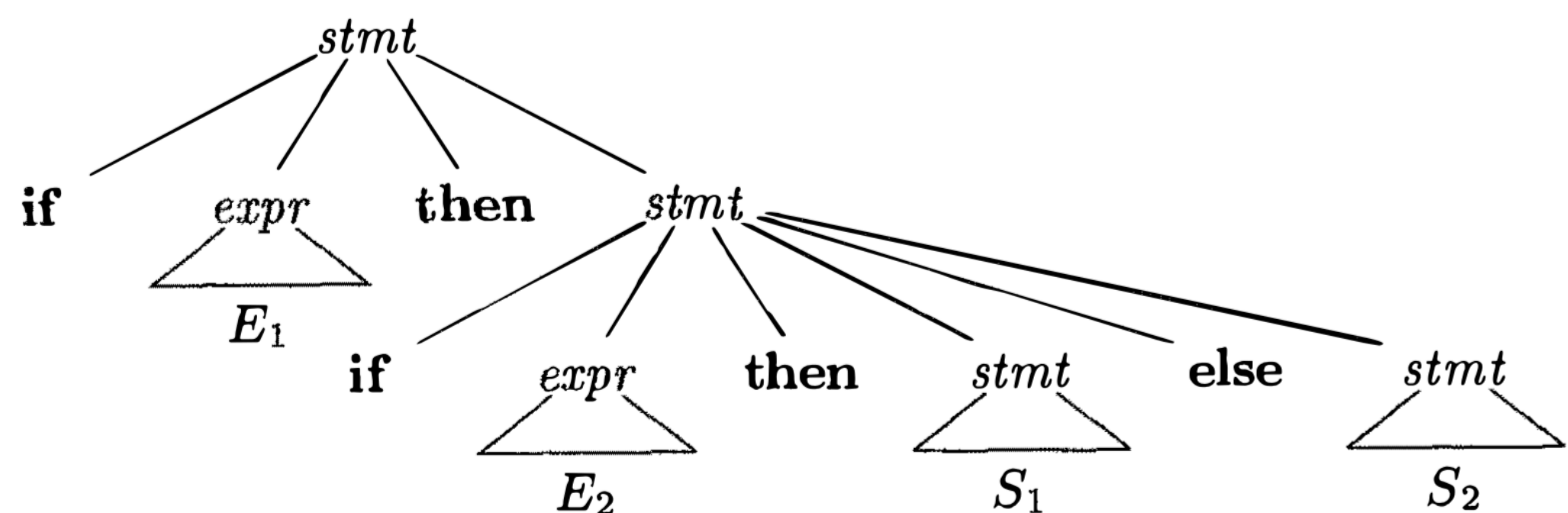
sorts S E

productions

S.If = if E then S

S.IfE = if E then S else S

S = other



derivation

$S \Rightarrow^*$ if E1 then if E2 then S1 else S2

term derivation

S

\Rightarrow If(E_1 , S)

\Rightarrow If(E_1 , IfE(E_2 , S_1 , S_2))

derivation

S

\Rightarrow if E1 then S

\Rightarrow if E1 then if E2 then S1 else S2

term derivation

S

\Rightarrow IfE(E_1 , S, S_2)

\Rightarrow IfE(E_1 , If(E_2 , S_1), S_2)

derivation

S

\Rightarrow if E1 then S else S2

\Rightarrow if E1 then if E2 then S1 else S2

Eliminating Dangling Else Ambiguity

grammar

sorts S E

productions

S.If = if E then S

S.IfE = if E then S else S

S = other

grammar

productions

S.If = if E then S

S.IfE = if E then SE else S

S = other

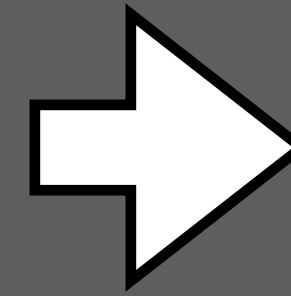
SE.IfE = if E then SE else SE

SE = other

Generalization of this transformation: contextual grammars

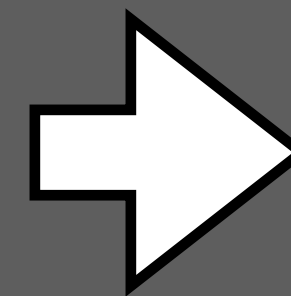
Eliminating Left Recursion

```
grammar
productions
E = E "+" T
E = T
T = T "*" F
T = F
F = "(" E ")"
F = ID
```



```
grammar
productions
E = T E'
E' = "+" T E'
E' =
T = F T'
T' = "*" F T'
T' =
F = "(" E ")"
F = ID
```

```
grammar
productions
A = A a
A = b
```

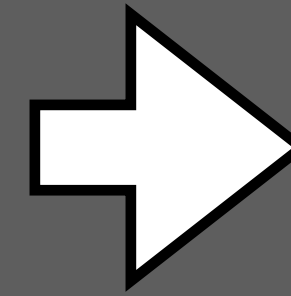


```
grammar
productions
A = b A'
A' = a A'
A' = // empty
```

// b followed by a list of as

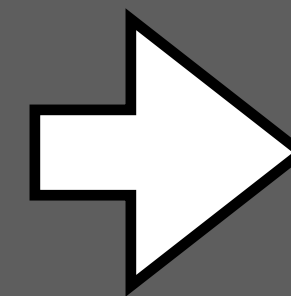
Eliminating Left Recursion using Regular Expressions

```
grammar
productions
E = E "+" T
E = T
T = T "*" F
T = F
F = "(" E ")"
F = ID
```



```
grammar
productions
E = T ("+" T)*
T = F ("*" F)*
F = "(" E ")"
F = ID
```

```
grammar
productions
A = A a
A = b
```



```
grammar
productions
A = b a*
```

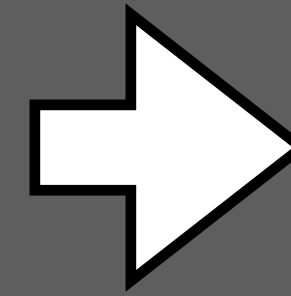
// b followed by a list of as

Left Factoring

grammar

productions

S.If = if E then S
S.IfE = if E then S else S
S = other



grammar

sorts S E

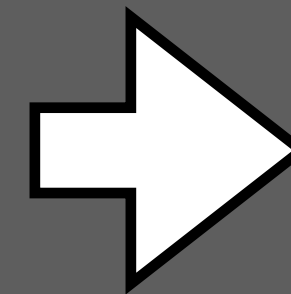
productions

S.If = if E then S S'
S'.Else = else S
S'.NoElse = // empty
S = other

grammar

productions

A = a b1
A = a b2
A = c



grammar

productions

A = a A'
A' = b1
A' = b2
A = c

Properties of Grammar Transformations

Preservation

- Preserves set of sentences
- Preserves set of trees
- Preserves tree structure

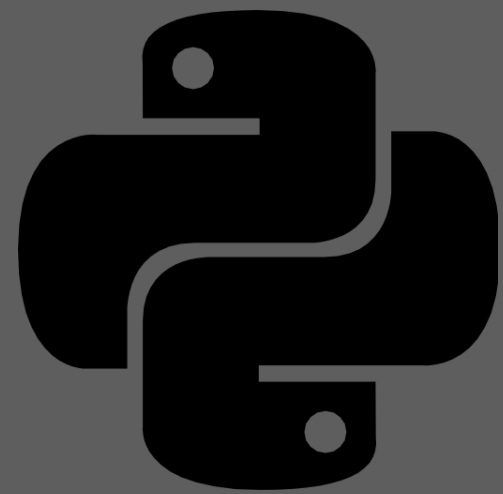
Systematic

- Algorithmic
- Heuristic

Layout-Sensitive Syntax

Layout-Sensitive Syntax

```
if x ≠ y:  
    if x > 0:  
        y = x  
else:  
    y = -x
```



```
guessValue x = do  
    putStrLn "Enter your guess:"  
    guess ← getLine  
    case compare (read guess) x of  
        EQ → putStrLn "You won!"  
        _  → do putStrLn "Keep guessing."  
              guessValue x
```



Disambiguation with Indentation Sensitive Context-free Grammars

```
case → 'case'> exp= 'of'> altBlock=  
-- Reset indentation for delimited blocks  
  
altBlock → '{'> alts* close*  
close → '}'>  
-- Increase indentation for non-delimited blocks  
  
altBlock → altLayout>  
altLayout → |alts≠ altLayout=  
altLayout → |alts≠  
-- Clause sequences  
  
alts → alt=  
alts → alt= ';'> alts=
```

Michael Adams. Principled Parsing for Indentation-Sensitive Languages: Revisiting Landin's Offside Rule. In POPL'13.

SDF3: Disambiguation with Layout Constraints

context-free syntax

```
Impl          = Stm {layout(1.first.col < 1.left.col)}  
Impls.StmSeq = Impl Impls {layout(1.first.col = 2.first.col)}  
  
Impls        = Impl  
Expls        = Stm  
Expls.StmtSeq = Stm ";" Expls  
Stms.Stms    = Impls  
  
Stms.Stms    = "{" Expls "}" {ignore-layout}
```

Sebastian Erdweg, Tillmann Rendel, Christian Kästner, and Klaus Ostermann.
Layout-Sensitive Generalized Parsing. In SLE'12.

Layout Constraints: Token Selectors

```
x = do 9 + 4
      * 3

main = do first putStrLn $
          show (x * right
                2) last
          left
```

```
lComp = do
  x <- xRange
  return $ do
    y <- yRange
    return (x, y)
```

Interpret indentation using shapes around the tokens that belong to a subtree.

Layout Constraints: Encoding Alignment

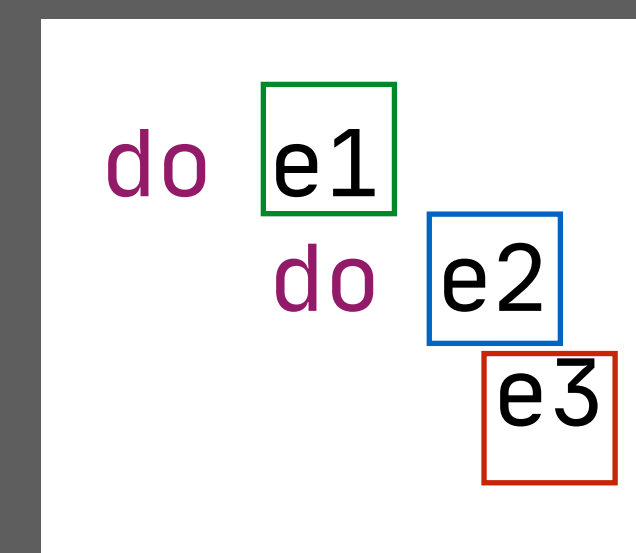
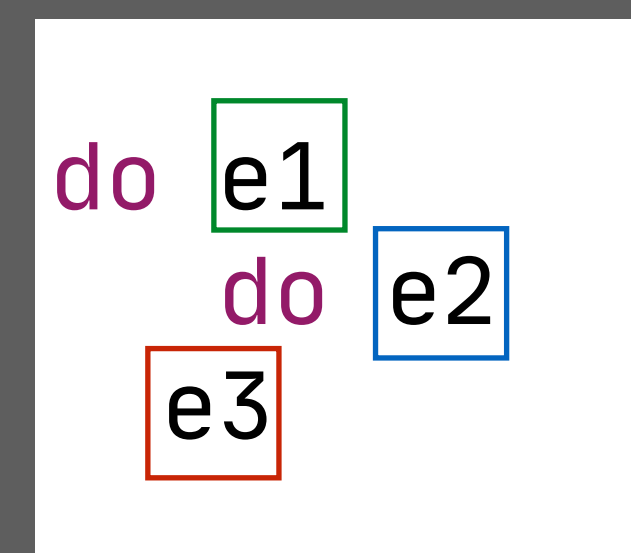
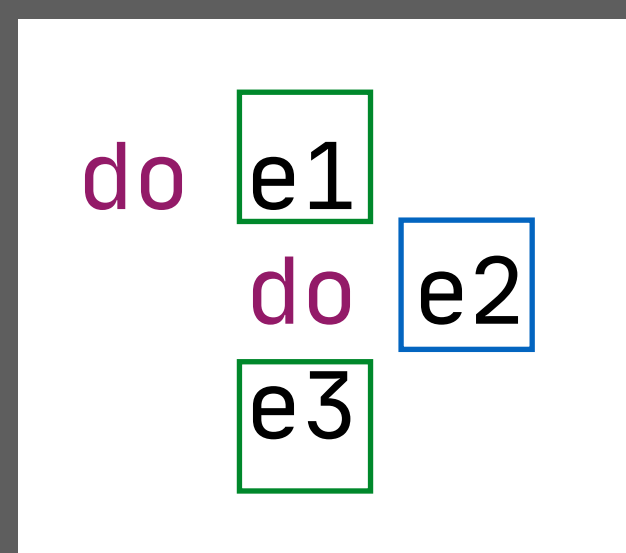
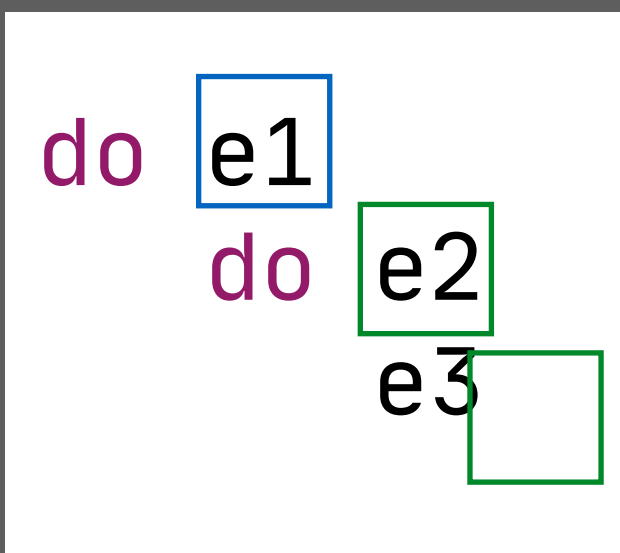
context-free syntax

```
Exp.Do      = "do" ExpList
```

```
ExpList.Cns = Exp
```

```
ExpList.Lst = ExpList Exp {layout(1.first.col = 2.first.col)}
```

```
Exp.Id      = ID
```



Layout Constraints: Encoding Indentation

```
Exp.IfElse = "if" Exp "then" Exp "else" Exp
  {layout(4.first.line > 1.last.line && // then clause in next line
         4.first.col > 1.first.col && // and indented
         1.first.col = 5.first.col && // "if" and "else" aligned
         4.first.col = 6.first.col)} // then and else clauses aligned
```

```
if e1 then
  if e2 then
    e3
  else
    e4
else
  e5
```

Low-level declarations

Layout Declarations

Tree Selectors

context-free syntax

```
Exp.Do      = "do" ExpList
ExpList.Cns = Exp
ExpList.Lst = ExpList Exp {layout(1.first.col = 2.first.col)}
Exp.Id      = ID
```

context-free syntax

```
Exp.Do      = "do" ExpList
ExpList.Cns = Exp
ExpList.Lst = exps:ExpList exp:Exp {layout(... exps exp)}
Exp.Id      = ID
```

Tree Selectors

context-free syntax

```
Exp.Do      = "do" ExpList {layout(2.first.col > 1.first.col)}  
ExpList.Cns = Exp  
ExpList.Lst = ExpList Exp {layout(1.first.col = 2.first.col)}  
Exp.Id      = ID
```

context-free syntax

```
Exp.Do      = "do" ExpList {layout(... "do" 1)}  
ExpList.Cns = Exp  
ExpList.Lst = exps:ExpList exp:Exp {layout(... exps exp)}  
Exp.Id      = ID
```

Alignment with Layout Constraints

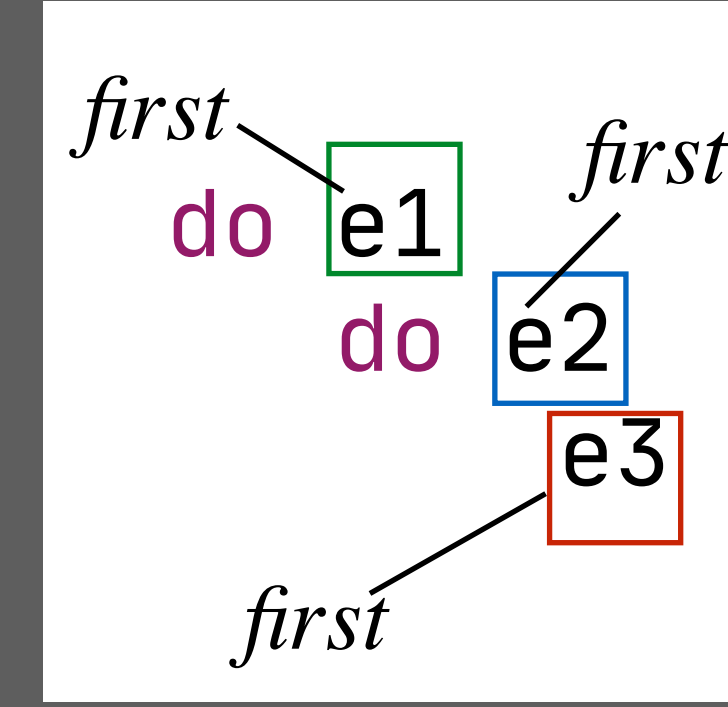
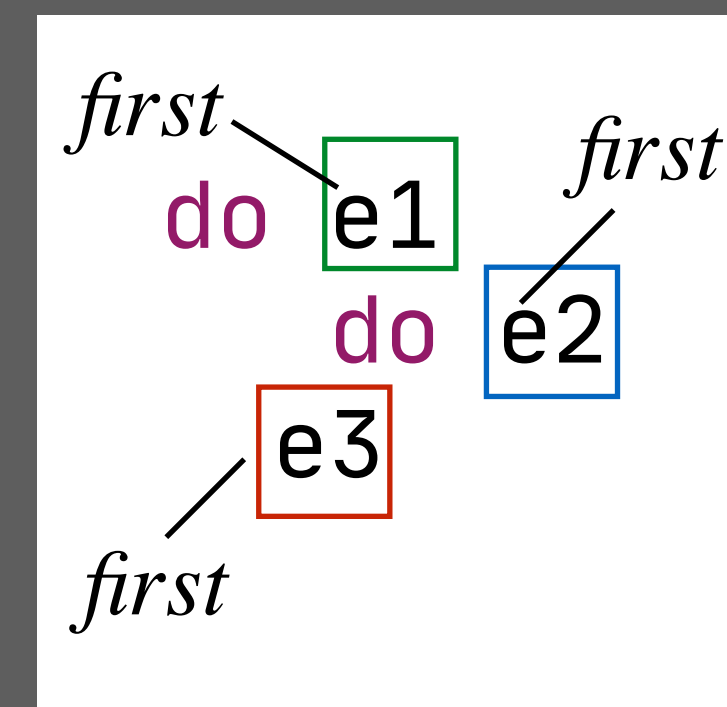
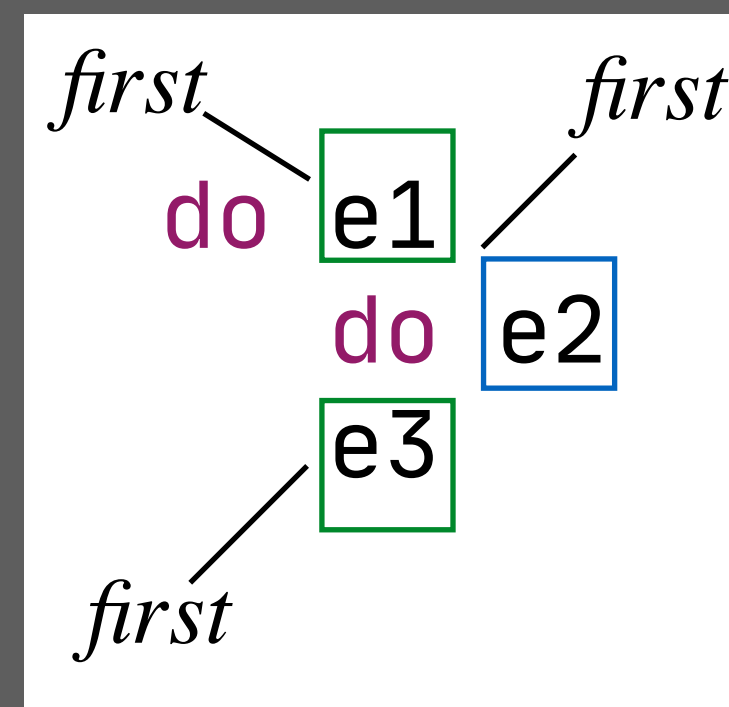
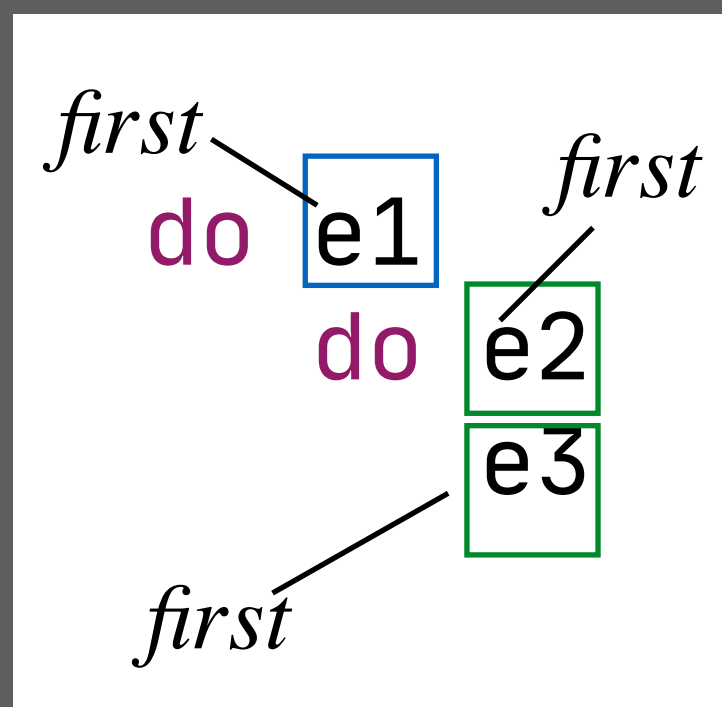
context-free syntax

Exp.Do = "do" ExpList

ExpList.Cns = Exp

ExpList.Lst = ExpList Exp {layout(1.first.col = 2.first.col)}

Exp.Id = ID



Alignment Declaration

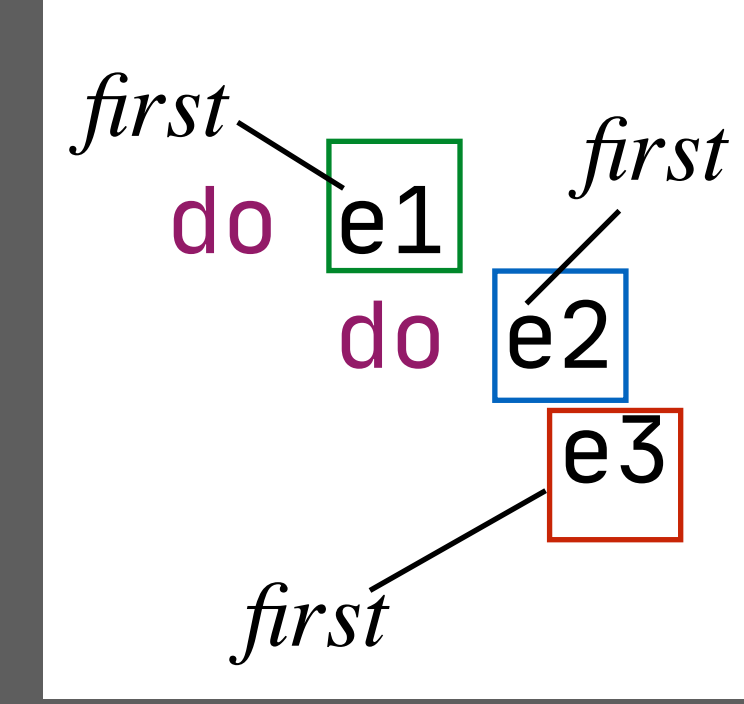
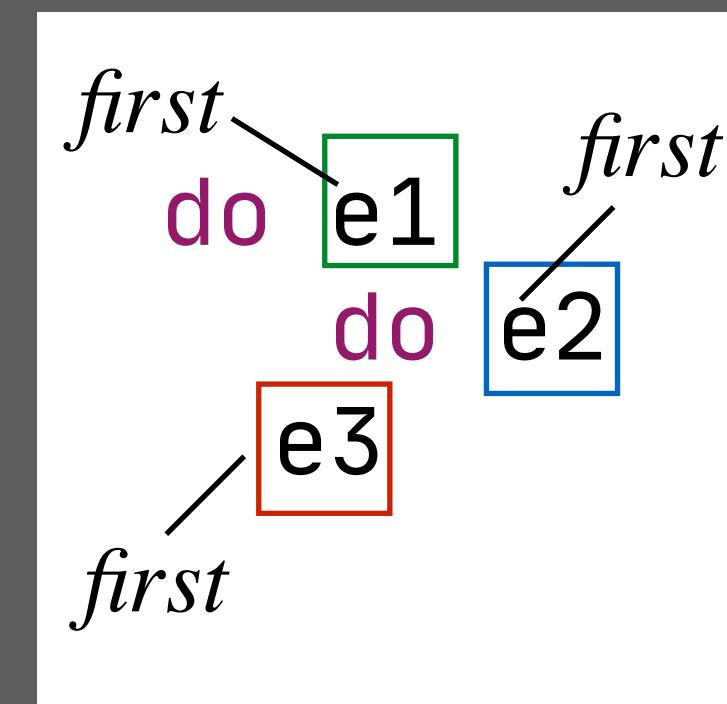
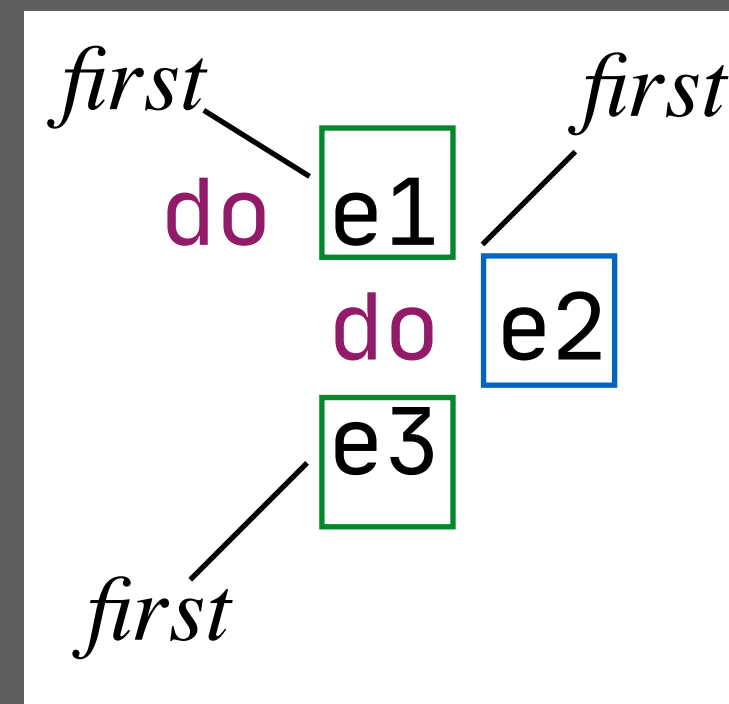
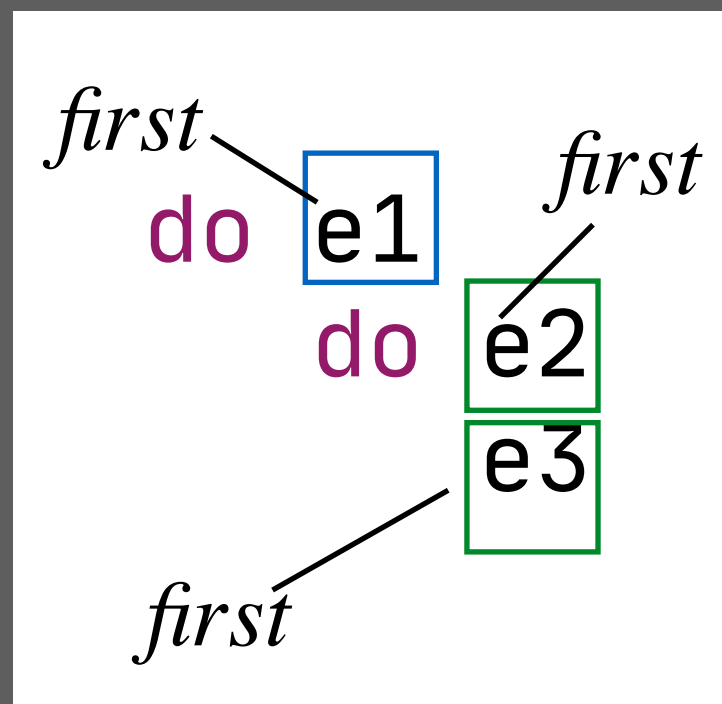
context-free syntax

Exp.Do = "do" ExpList

ExpList.Cns = Exp

ExpList.Lst = exps:ExpList exp:Exp {layout(align exps exp)}

Exp.Id = ID



Semantics

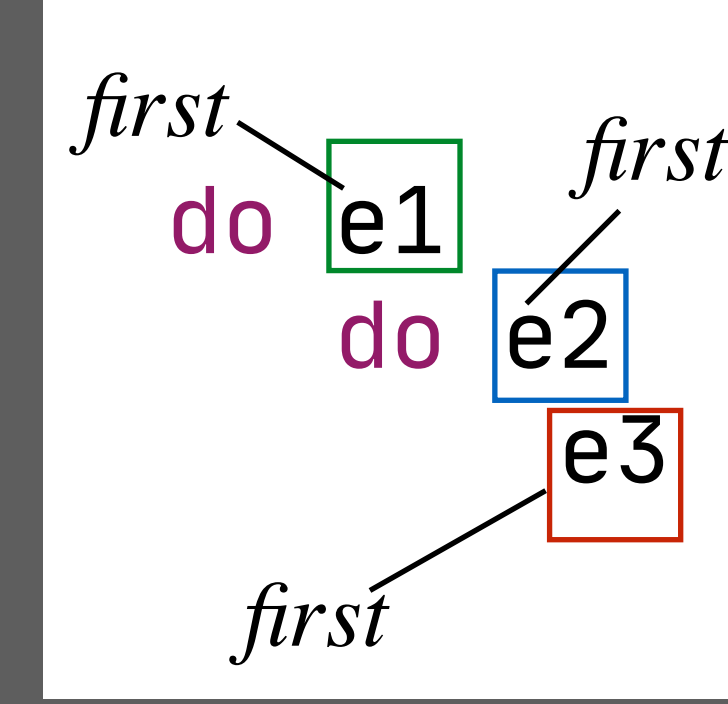
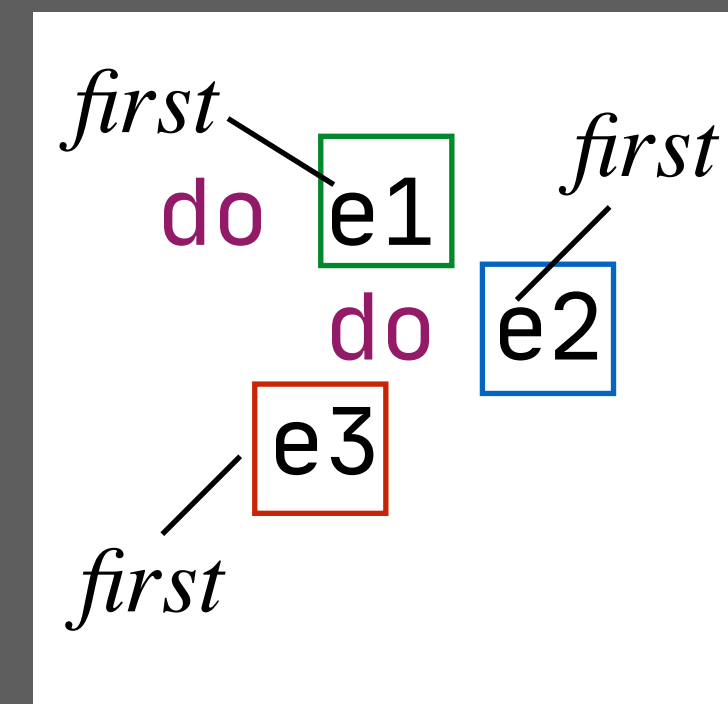
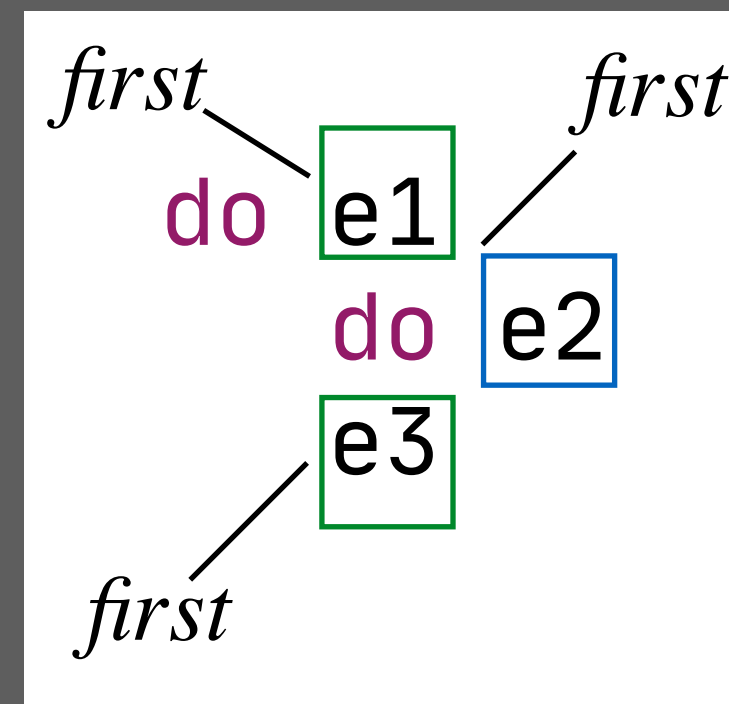
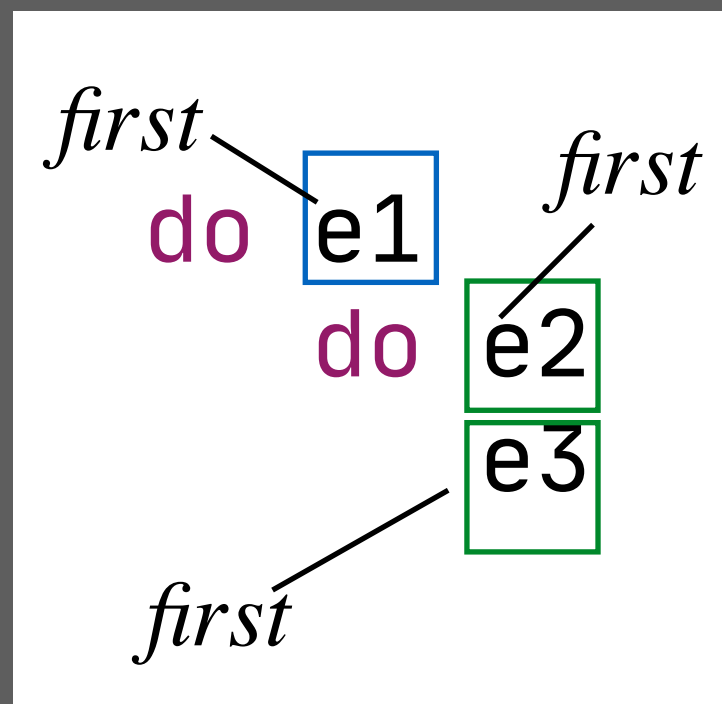
$x.first.col = y.first.col$

$align\ x\ y$

List Alignment Declaration

context-free syntax

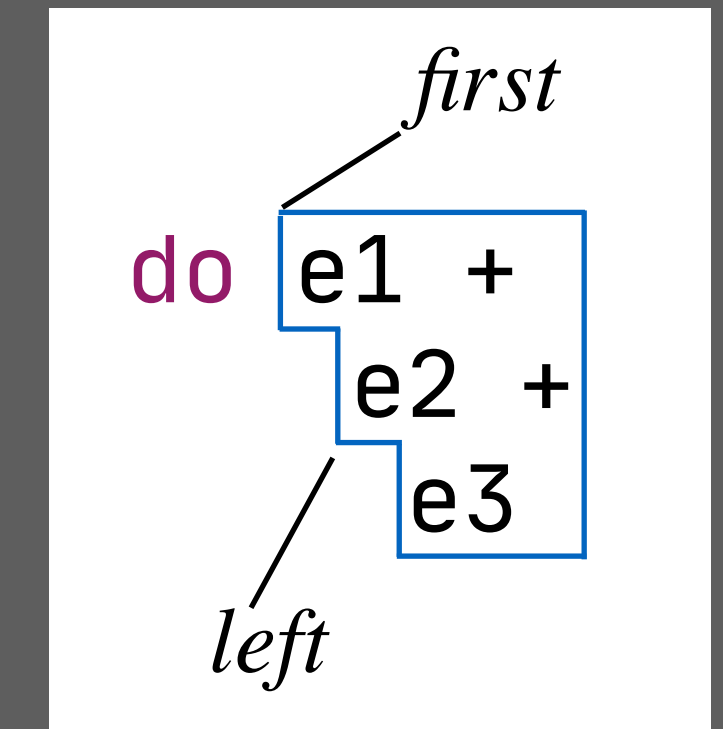
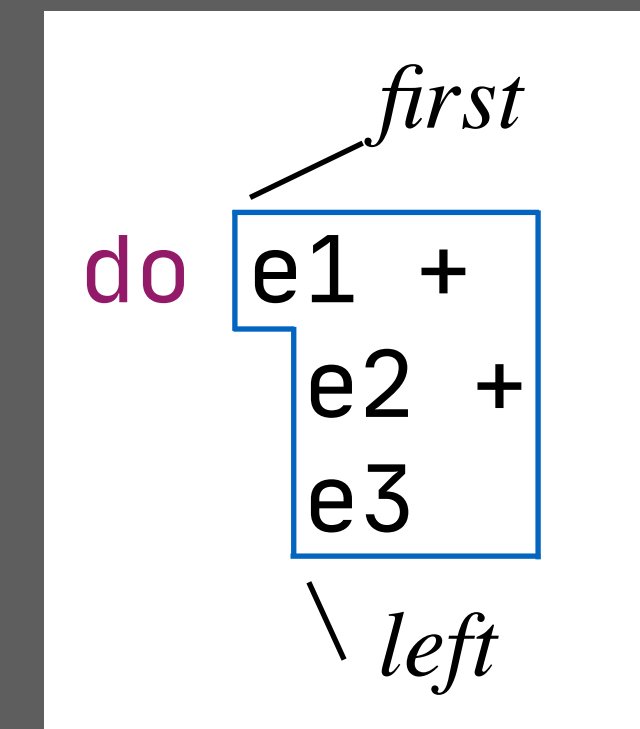
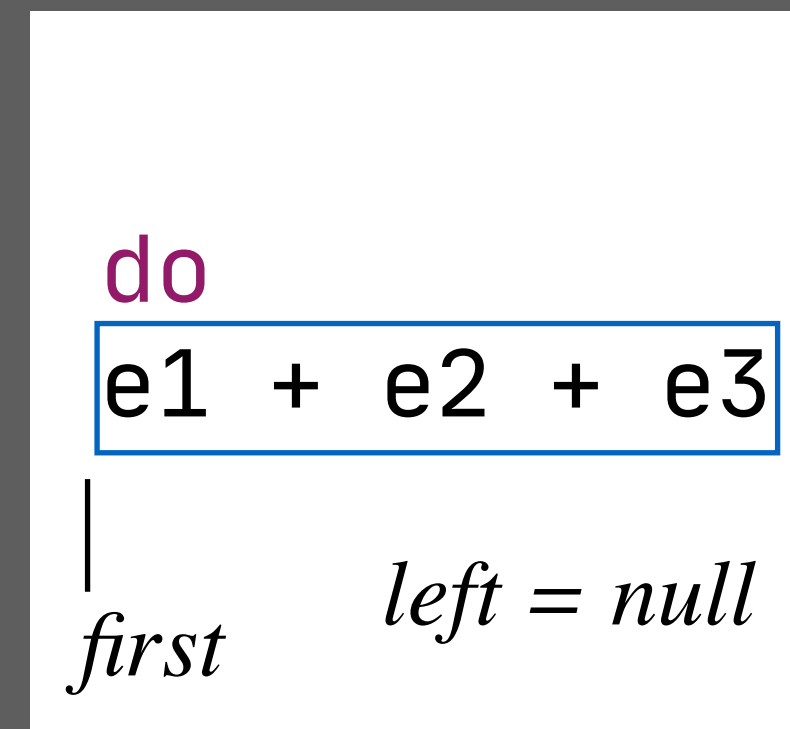
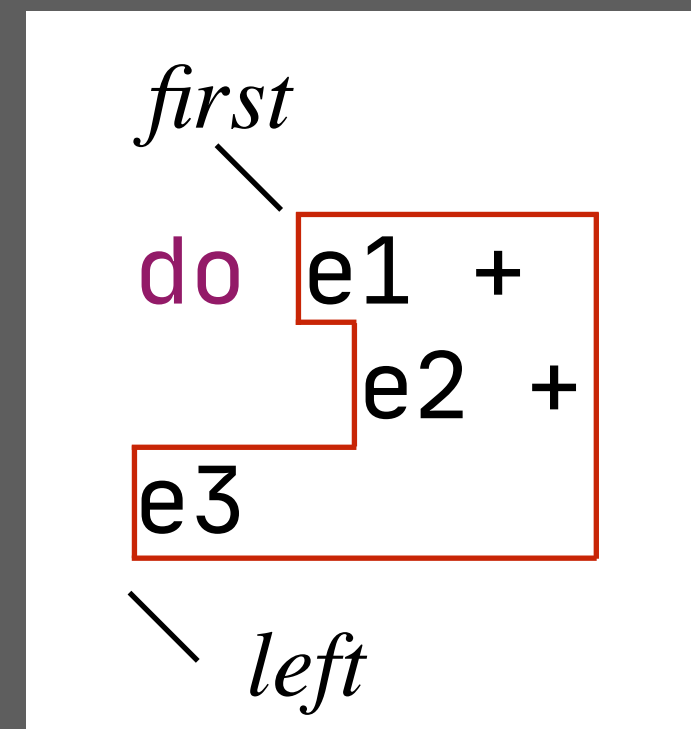
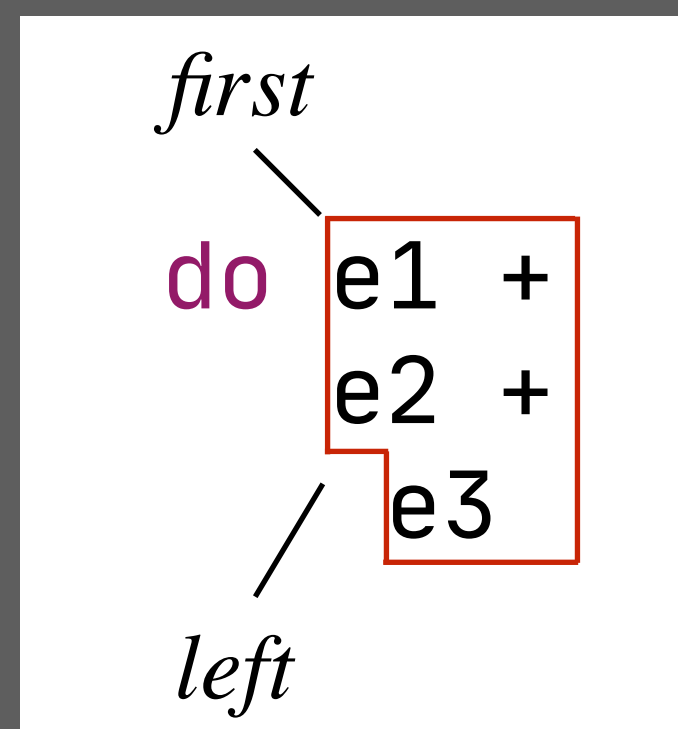
```
Exp.Do = "do" exps:Exp+ {layout(align-list exps)}  
Exp.Id = ID  
Exp+ = Exp+ Exp // normalized  
Exp+ = Exp // productions
```



Semantics

$$\frac{A+ = A+ A \text{ layout}(1.\text{first.col} = 2.\text{first.col})}{\text{align-list } x}$$

Offside Rule



“The offside rule prescribes that all non-whitespace tokens of a structure must be further to the right than the token that starts the structure.”

Erdweg et. al.. Layout-Sensitive Generalized Parsing. In SLE'12.

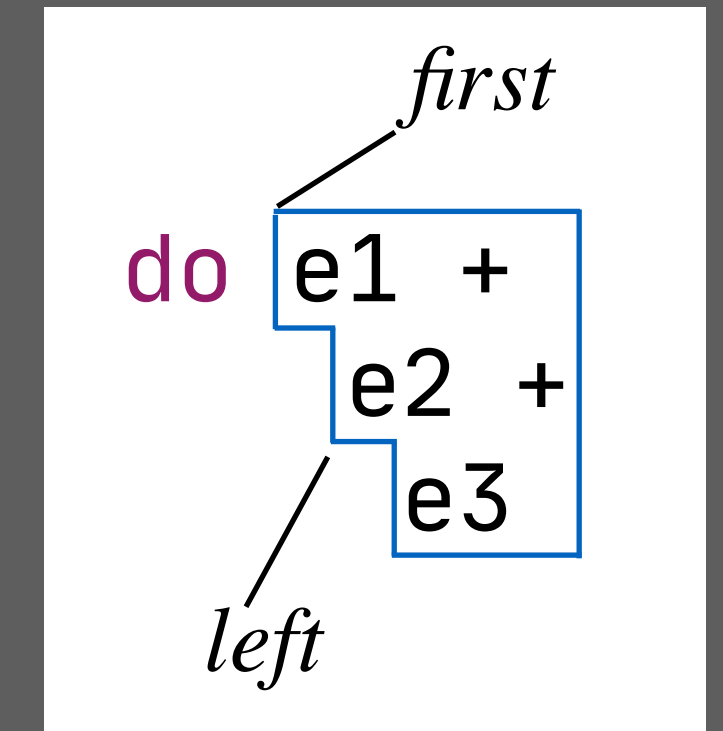
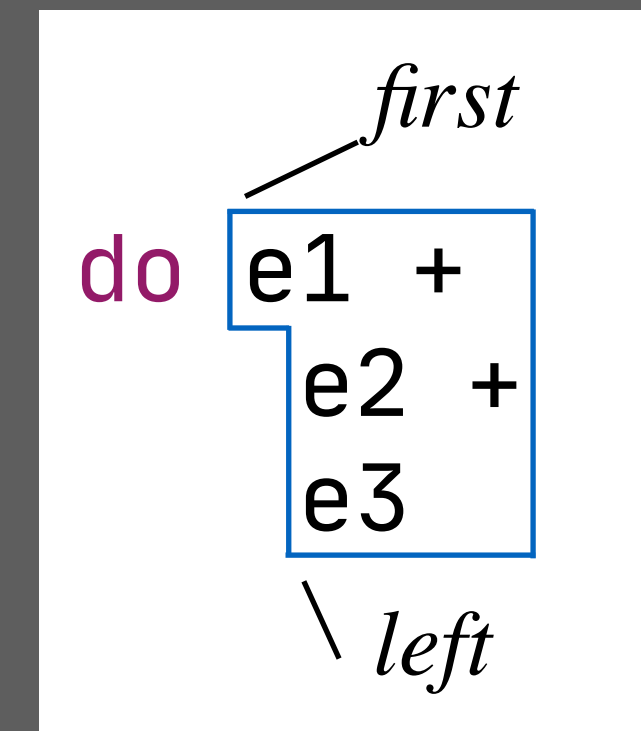
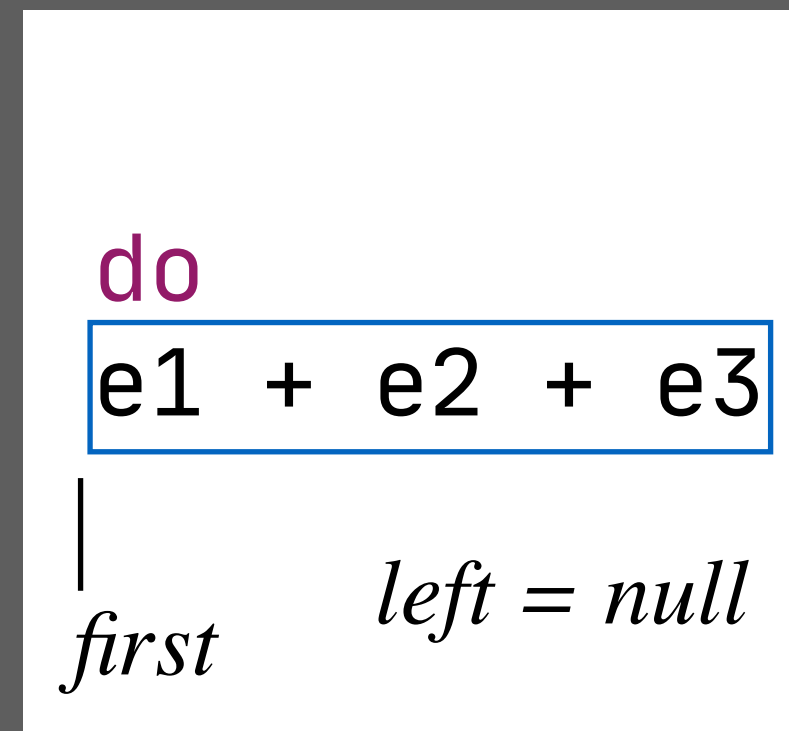
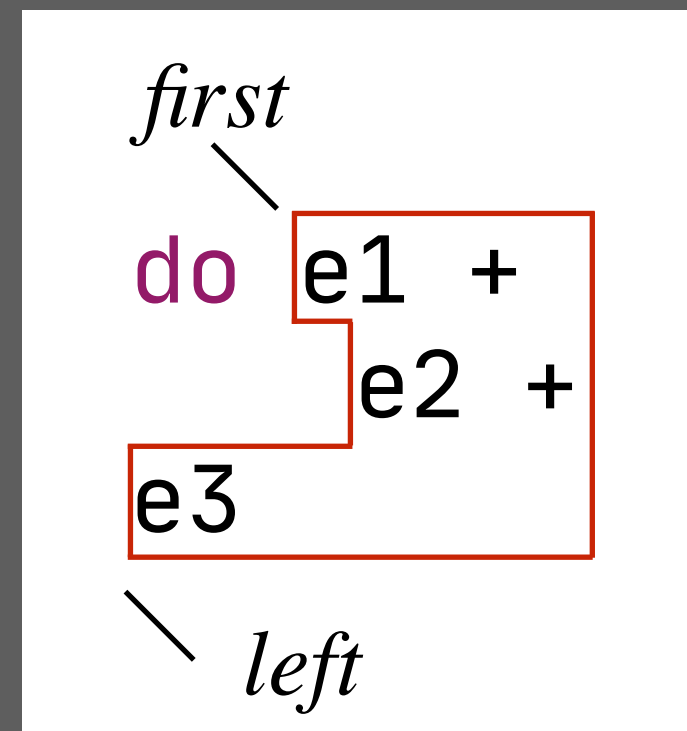
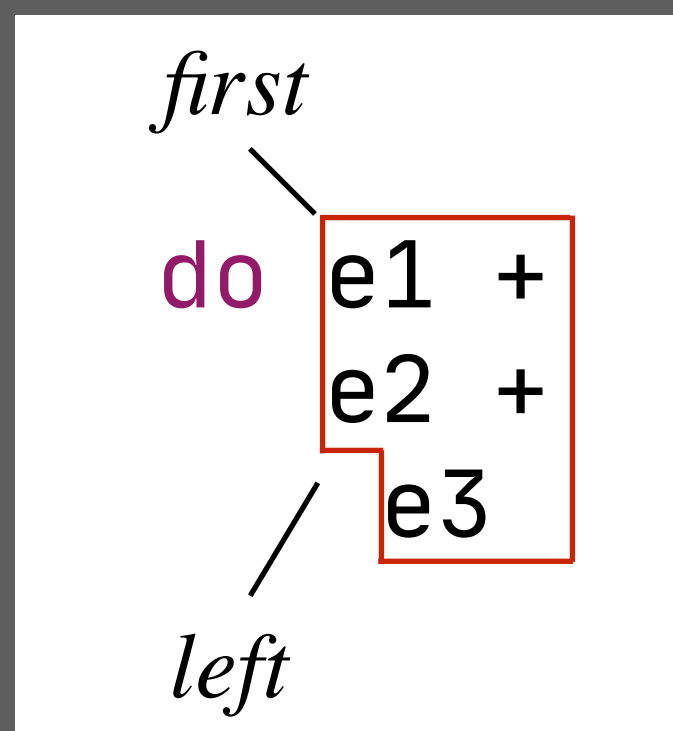
Offside with Layout Constraints

context-free syntax

Exp.Do = "do" Exp {layout(2.left.col > 2.first.col)}

Exp.Add = Exp "+" Exp {left}

Exp.Id = ID



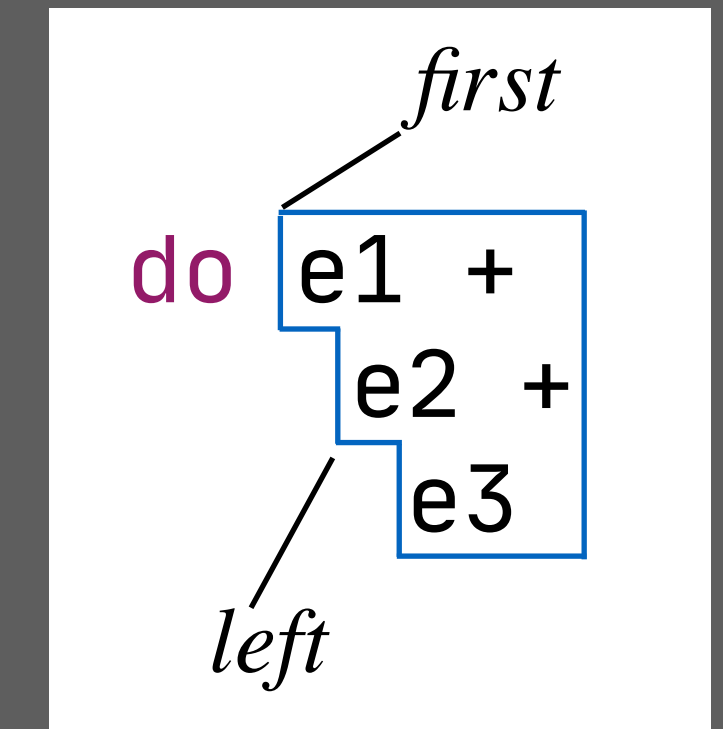
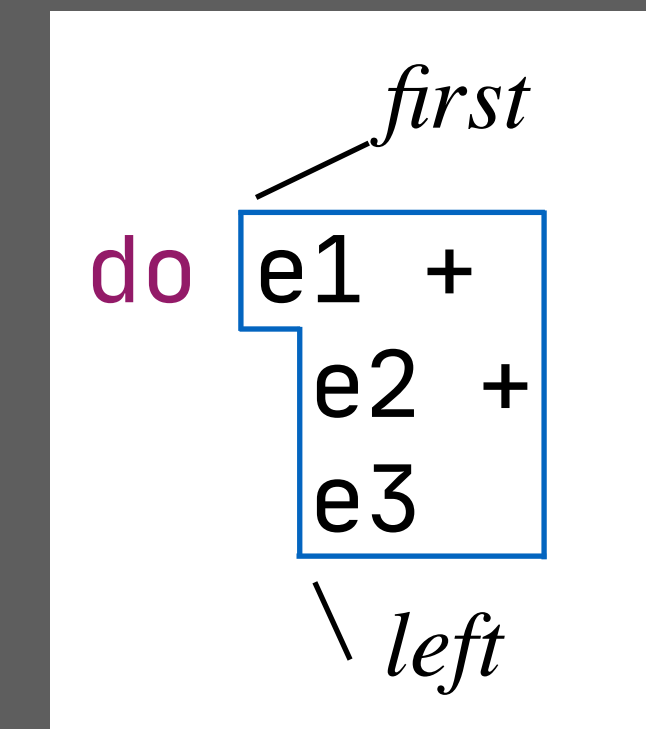
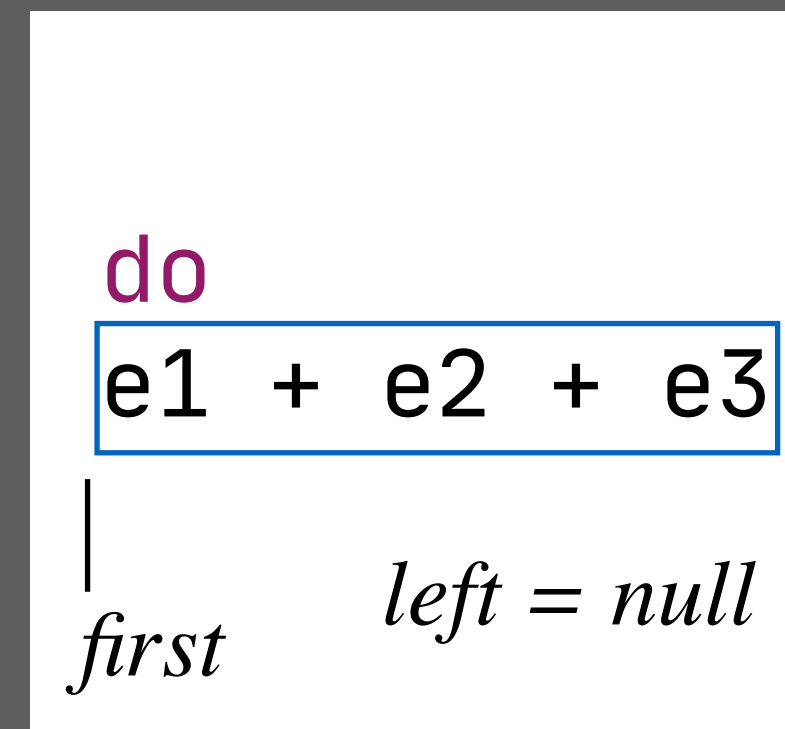
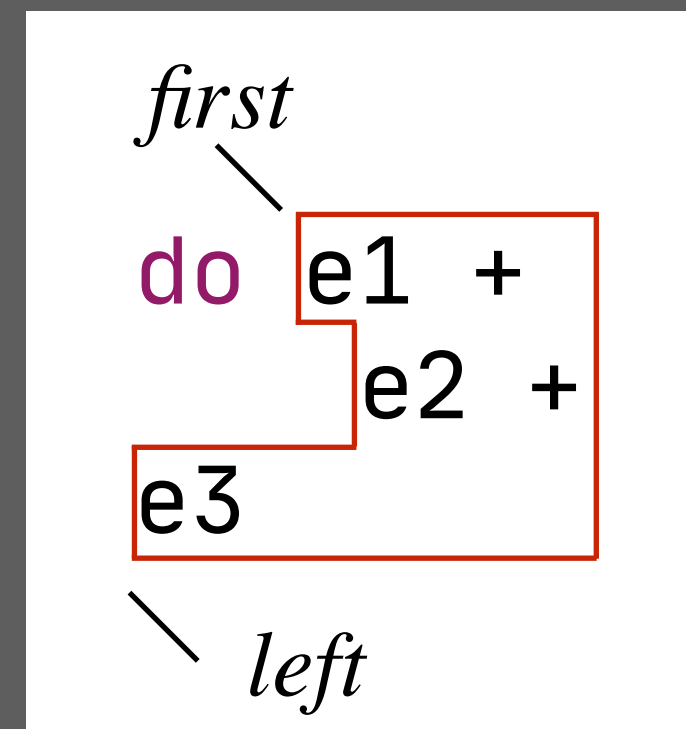
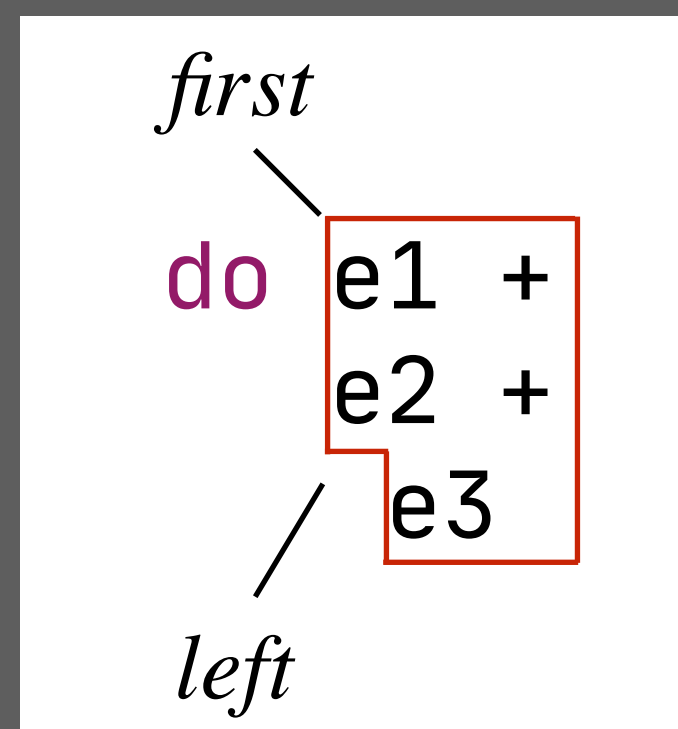
Offside

context-free syntax

Exp.Do = "do" exp:Exp {layout(offside exp)}

Exp.Add = Exp "+" Exp {left}

Exp.Id = ID



Semantics

$x.\text{left.col} > x.\text{first.col}$

offside x

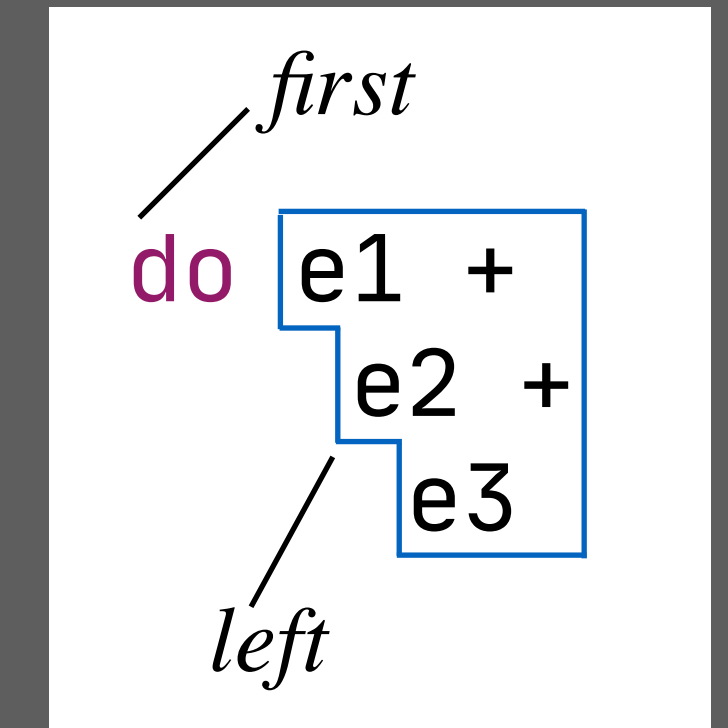
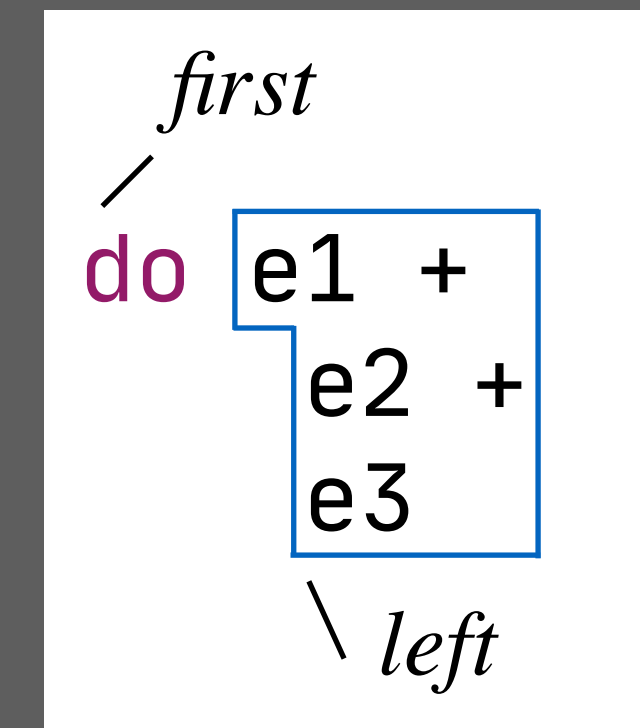
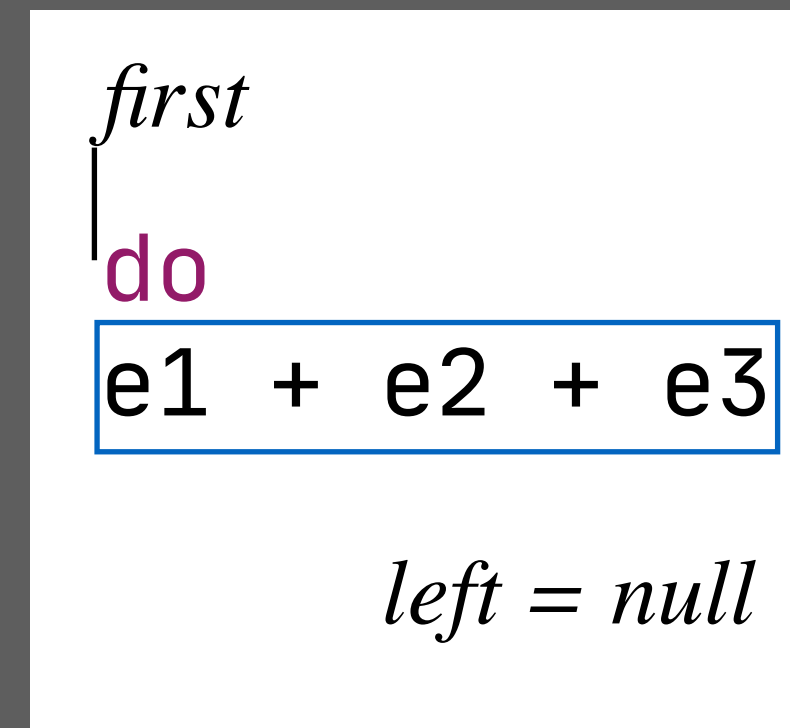
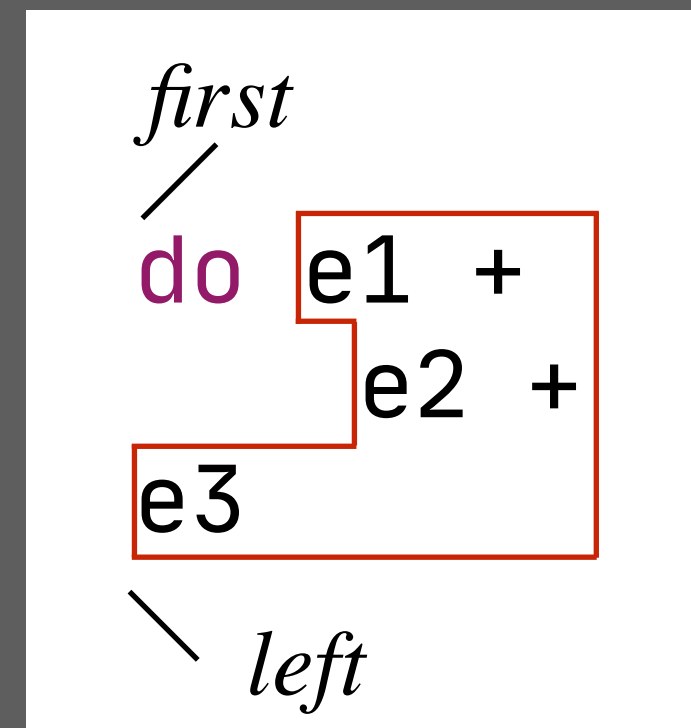
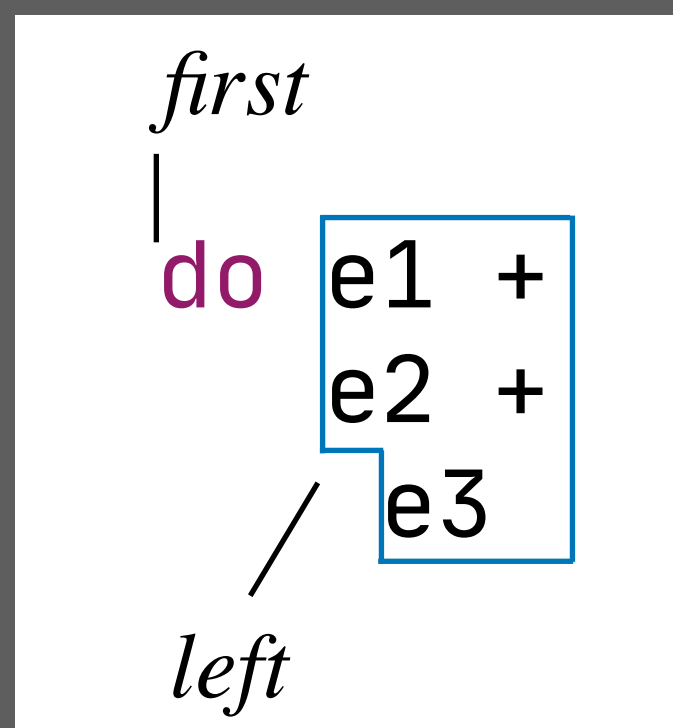
Relative Offside

context-free syntax

Exp.Do = "do" exp:Exp {layout(offside "do" exp)}

Exp.Add = Exp "+" Exp {left}

Exp.Id = ID



Semantics

$$\frac{y.\text{left.col} > x.\text{first.col}}{\text{offside } x \ y}$$

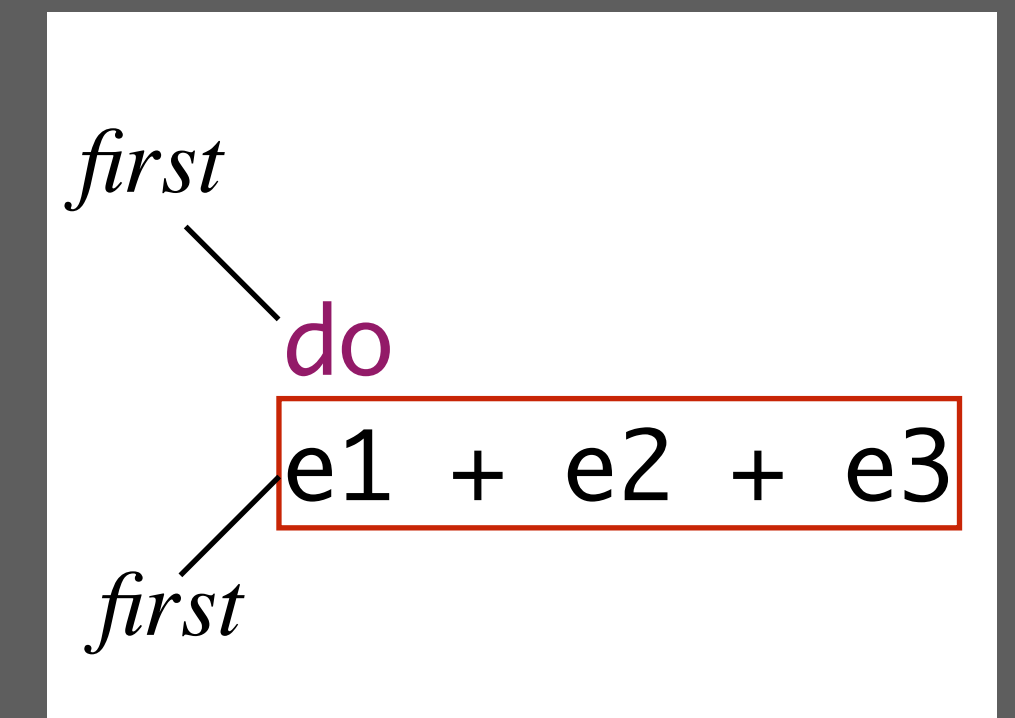
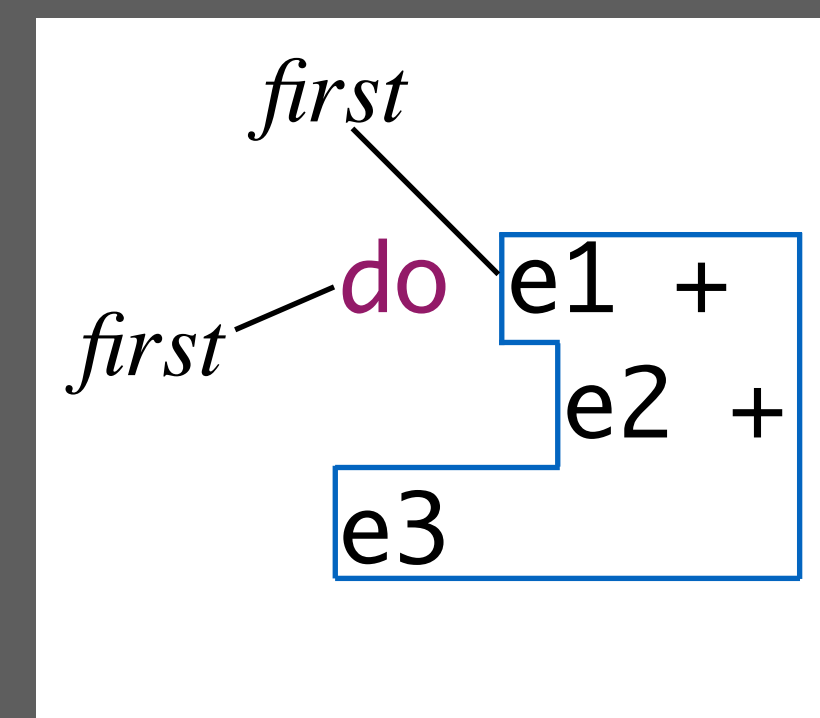
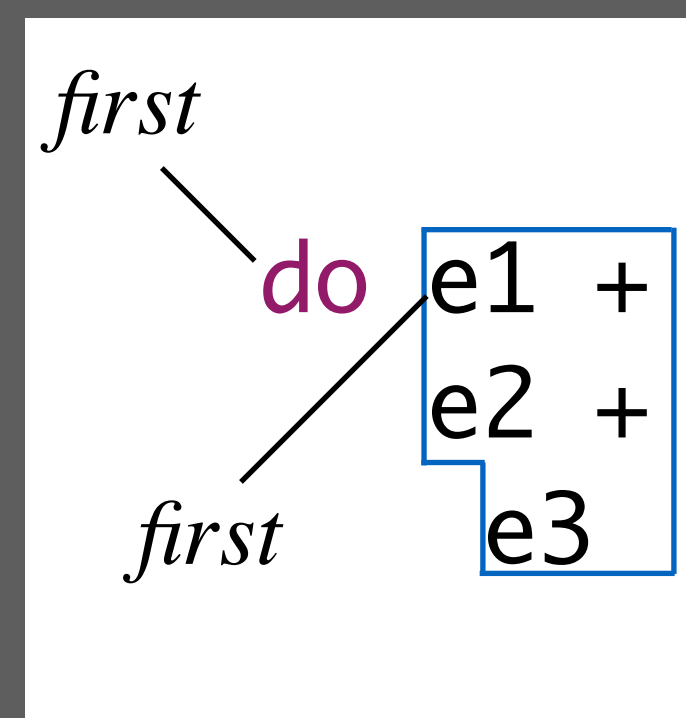
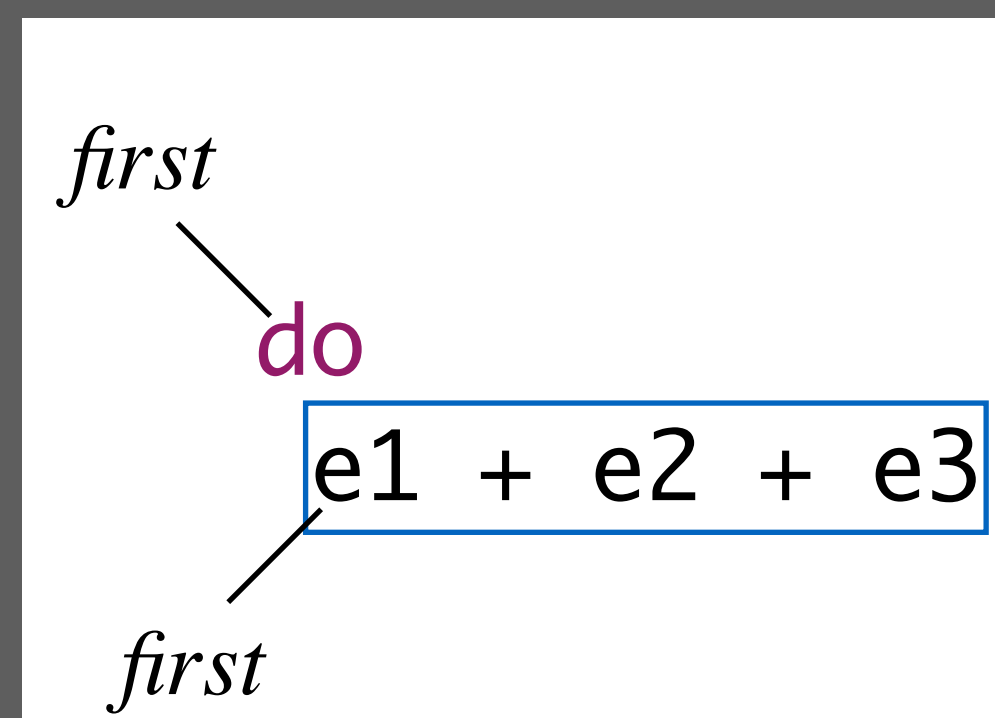
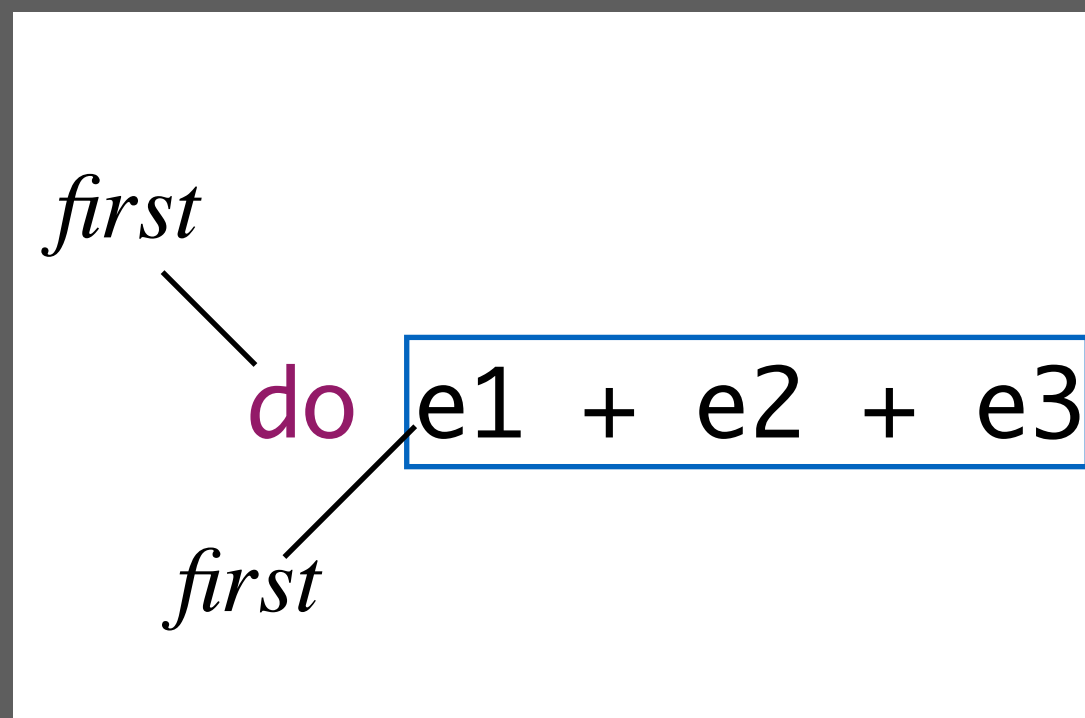
Indentation

context-free syntax

Exp.Do = "do" exp:Exp {layout(indent "do" exp)}

Exp.Add = Exp "+" Exp {left}

Exp.Id = ID



Semantics

$y.first.col > x.first.col$

$indent\ x\ y$

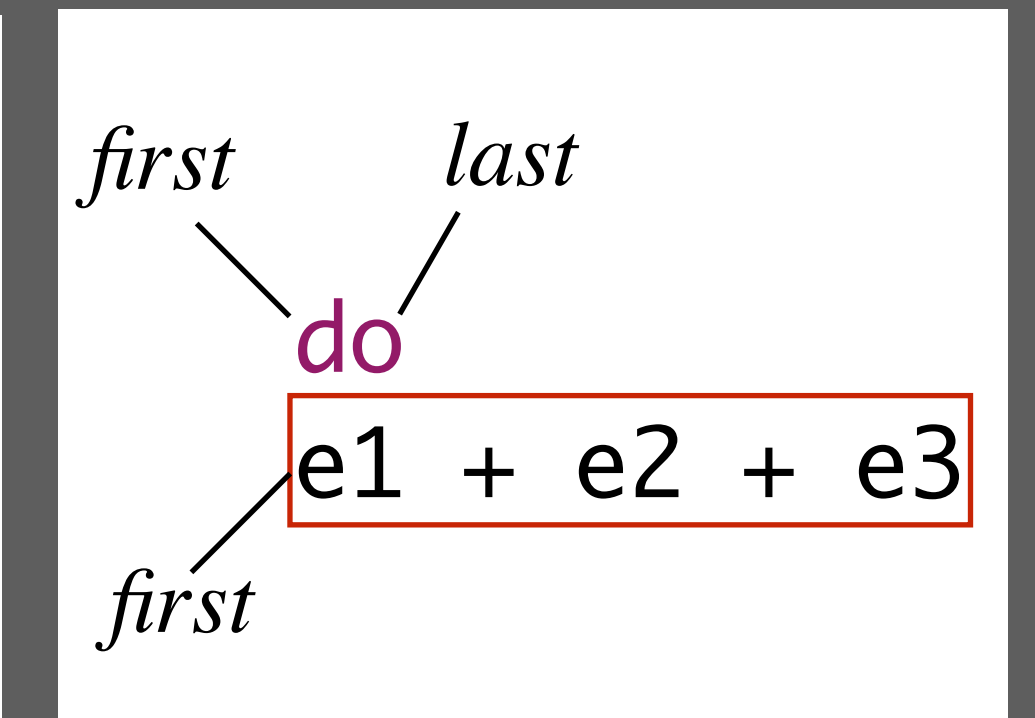
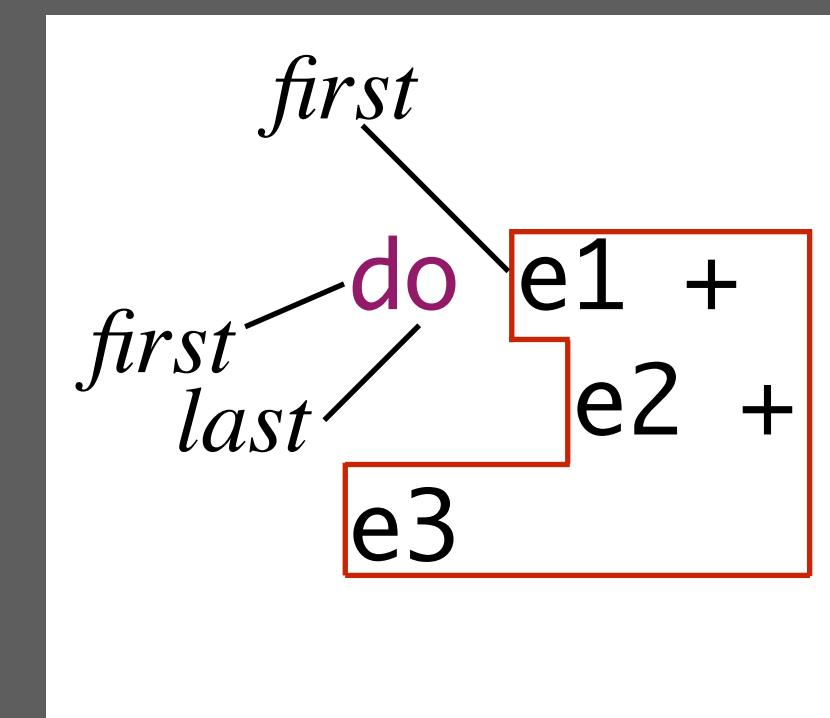
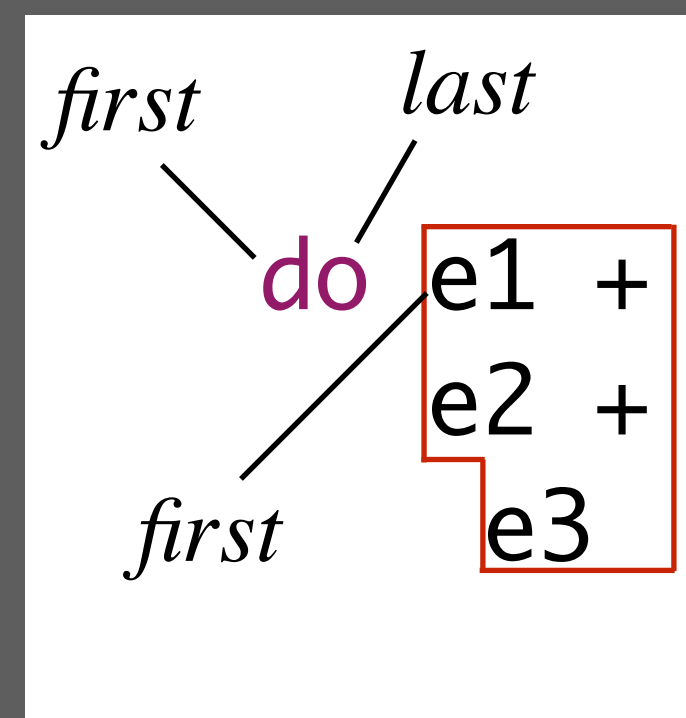
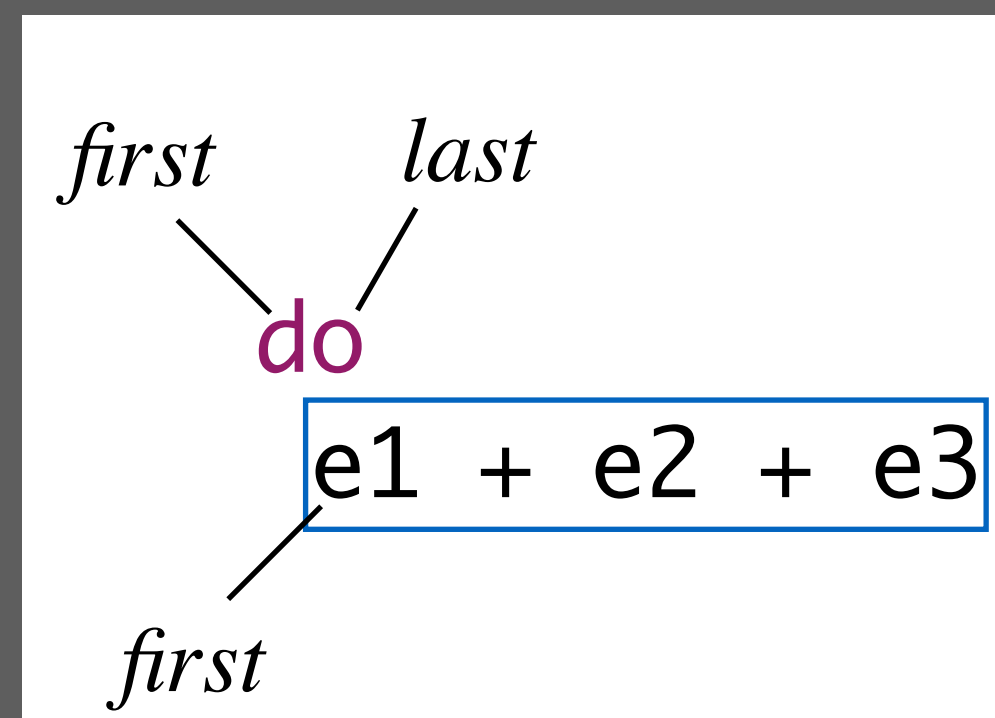
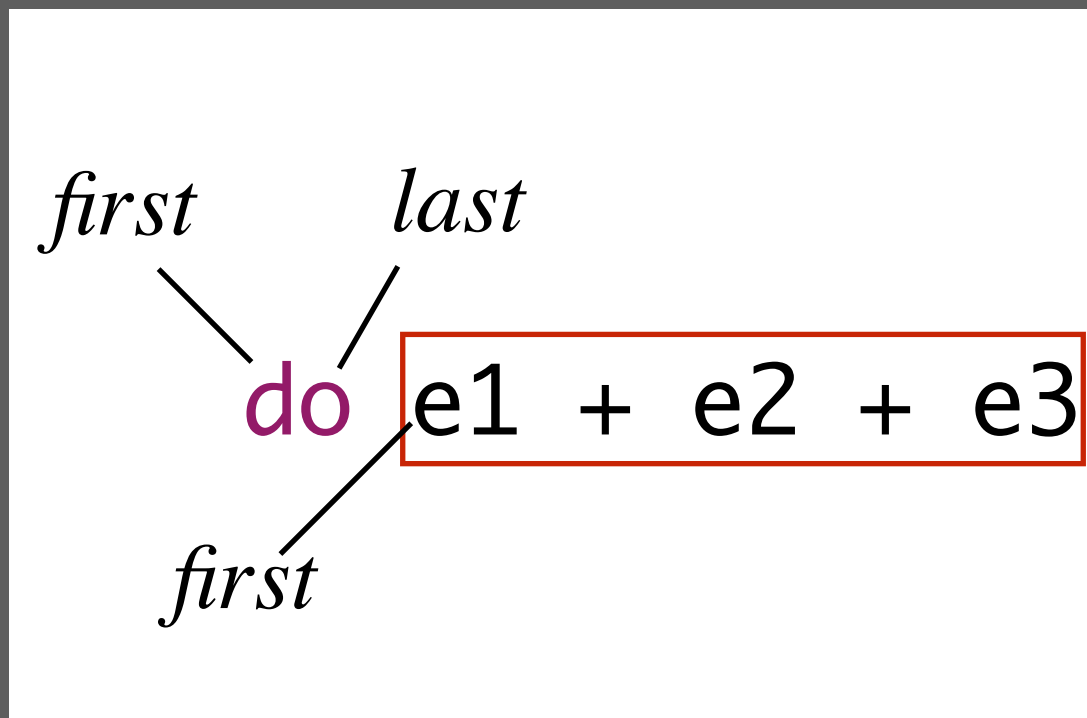
Newline + Indentation

context-free syntax

Exp.Do = "do" exp:Exp {layout(newline-indent "do" exp)}

Exp.Add = Exp "+" Exp {left}

Exp.Id = ID



Semantics

$y.first.col > x.first.col \ \&\& \ y.first.line > x.last.line$

newline-indent x y

Parsing and Pretty-Printing

runtime-Spoofax - LayoutSens/syntax/LayoutSens.sdf3 - Eclipse IDE

```
LayoutSens.sdf3
1 module LayoutSens
2
3 imports Common
4
5 context-free syntax
6
7 Exp.Do = "do" stmts:Stmt+
8   {layout(align-list stmts),
9    longest-match}
10
11 Stmt.OffsideExp = exp:Exp
12   {layout(offside exp)}
13
14 Exp.IfElse = "if" Exp "then" then:Exp "
15   {layout(pp-newline-indent "if" then &
16    pp-align "if" "else" &&
17    align then else )}
18
19 Stmt.Assign = ID "=" Exp
20 Exp.Add = Exp "+" Exp {left}
21 Exp = INT
22 Exp = ID
23
24 context-free priorities
25 Exp.Add > Exp.Do > Exp.IfElse
26
27 lexical syntax
28
29 Exp.GenericExp = "e" INT
30 Stmt.GenericExp = "s" INT
```

```
example1.lsn
1 if e1 then if e2 then e3
2   else e4
3 else e5
4
5 do x = 1
6   y = 2
7   x
8   + y
```

Parse

```
example1.aterm
1 Exps(
2   [ IfElse("e1", IfElse("e2", "e3", "e4")
3     , Do([Assign("x", "1"), Assign("y", "2")
4       , Add("x", "y")
5     ]
6 )
```

Pretty-Print

```
example1.pp.lsn
1 if e1 then
2   if e2 then
3     e3
4   else
5     e4
6 else
7   e5
8
9 do x = 1
10  y = 2
11
12 x + y
```

Writable Insert 1:1

{S} spoofax

Reading Material

Introduces layout constraints

Layout-Sensitive Generalized Parsing

Sebastian Erdweg, Tillmann Rendel, Christian Kästner, and Klaus Ostermann

University of Marburg, Germany

Abstract. The theory of context-free languages is well-understood and context-free parsers can be used as off-the-shelf tools in practice. In particular, to use a context-free parser framework, a user does not need to understand its internals but can specify a language *declaratively* as a grammar. However, many languages in practice are not context-free. One particularly important class of such languages is layout-sensitive languages, in which the structure of code depends on indentation and whitespace. For example, Python, Haskell, F#, and Markdown use indentation instead of curly braces to determine the block structure of code. Their parsers (and lexers) are not declaratively specified but hand-tuned to account for layout-sensitivity.

To support *declarative* specifications of layout-sensitive languages, we propose a parsing framework in which a user can annotate layout in a grammar. Annotations take the form of constraints on the relative positioning of tokens in the parsed subtrees. For example, a user can declare that a block consists of statements that all start on the same column. We have integrated layout constraints into SDF and implemented a layout-sensitive generalized parser as an extension of generalized LR parsing. We evaluate the correctness and performance of our parser by parsing 33 290 open-source Haskell files. Layout-sensitive generalized parsing is easy to use, and its performance overhead compared to layout-insensitive parsing is small enough for practical application.

1 Introduction

Most computer languages prescribe a textual syntax. A parser translates from such textual representation into a structured one and constitutes the first step in processing a document. Due to the development of parser frameworks such as lex/yacc [15], ANTLR [18,17], PEGs [6,7], parsec [13], or SDF [8], parsers can be considered off-the-shelf tools nowadays: Non-experts can use parsers, because language specifications are declarative. Although many parser frameworks support some form of context-sensitive parsing (such as via semantic predicates in ANTLR [18]), one particularly relevant class of languages is not supported declaratively by any existing parser framework: layout-sensitive languages.

Layout-sensitive languages were proposed by Landin in 1966 [12]. In layout-sensitive languages, the translation from a textual representation to a structural one depends on the code's layout and its indentation. Most prominently, the *off-side rule* prescribes that all non-whitespace tokens of a structure must be further to the right than the token that starts the structure. In other words, a token

https://doi.org/10.1007/978-3-642-36089-3_14

Introduces layout declarations to abstract from low-level layout constraints and pretty-printing based on layout declarations/constraints.

Includes summary of layout constraints

Won distinguished paper award at SLE'18.

<https://doi.org/10.1145/3276604.3276607>



Declarative Specification of Indentation Rules

A Tooling Perspective on Parsing and Pretty-Printing Layout-Sensitive Languages

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Abstract

In layout-sensitive languages, the indentation of an expression or statement can influence how a program is parsed. While some of these languages (e.g., Haskell and Python) have been widely adopted, there is little support for software language engineers in building tools for layout-sensitive languages. As a result, parsers, pretty-printers, program analyses, and refactoring tools often need to be handwritten, which decreases the maintainability and extensibility of these tools. Even state-of-the-art language workbenches have little support for layout-sensitive languages, restricting the development and prototyping of such languages.

In this paper, we introduce a novel approach to declarative specification of layout-sensitive languages using *layout declarations*. Layout declarations are high-level specifications of indentation rules that abstract from low-level technicalities. We show how to derive an efficient layout-sensitive generalized parser and a corresponding pretty-printer automatically from a language specification with layout declarations. We validate our approach in a case-study using a syntax definition for the Haskell programming language, investigating the performance of the generated parser and the correctness of the generated pretty-printer against 22191 Haskell files.

CCS Concepts • Software and its engineering → Syntax; Parsers;

Keywords parsing, pretty-printing, layout-sensitivity

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ACM Reference Format:

Luís Eduardo de Souza Amorim, Michael J. Steindorfer, Sebastian Erdweg, and Eelco Visser. 2018. Declarative Specification of Indentation Rules: A Tooling Perspective on Parsing and Pretty-Printing Layout-Sensitive Languages. In *Proceedings of the 11th ACM SIGPLAN International Conference on Software Language Engineering (SLE '18)*, November 5–6, 2018, Boston, MA, USA. ACM, New York, NY, USA, 13 pages. <https://doi.org/10.1145/3276604.3276607>

1 Introduction

Layout-sensitive (also known as indentation-sensitive) languages were introduced by Landin [17]. The term characterizes languages that must obey certain *indentation rules*, i.e., languages in which the indentation of the code influences how the program should be parsed. In layout-sensitive languages, alignment and indentation are essential to correctly identify the structures of a program. Many modern programming languages including Haskell [11], Python [23], Markdown [14] and YAML [4] are layout-sensitive. To illustrate how layout can influence parsing programs in such languages, consider the Haskell program in Figure 1, which contains multiple *do*-expressions:

```
1 guessValue x = do
2   putStrLn "Enter your guess:"
3   guess <- getLine
4   case compare (read guess) x of
5     EQ -> putStrLn "You won!"
6     _  -> do putStrLn "Keep guessing."
7           guessValue x
```

Figure 1. *Do*-expressions in Haskell.

In Haskell, all statements inside a *do*-block should be aligned (i.e., should start at the same column). In Figure 1, we know that the statement on line 7 (`guessValue x`) belongs to the inner *do*-block solely because of its indentation. If we modify the indentation of this statement, aligning it with the statements in the outer *do*-block, the program would have a different interpretation, looping indefinitely.

Next: Static Analysis and Type Checking